

Exemplar 13 : Solving Simultaneous Linear Equations Graphically

Objective: To solve simultaneous linear equations by using a graphing software

Key Stage: 3

Learning Unit: Linear Equations in Two Unknowns

Material Required: Graphing software - *Graphmatica*

Prerequisite Knowledge:

- (1) The graphical method in solving simultaneous linear equations
- (2) Rounding off numbers to the required number of decimal places

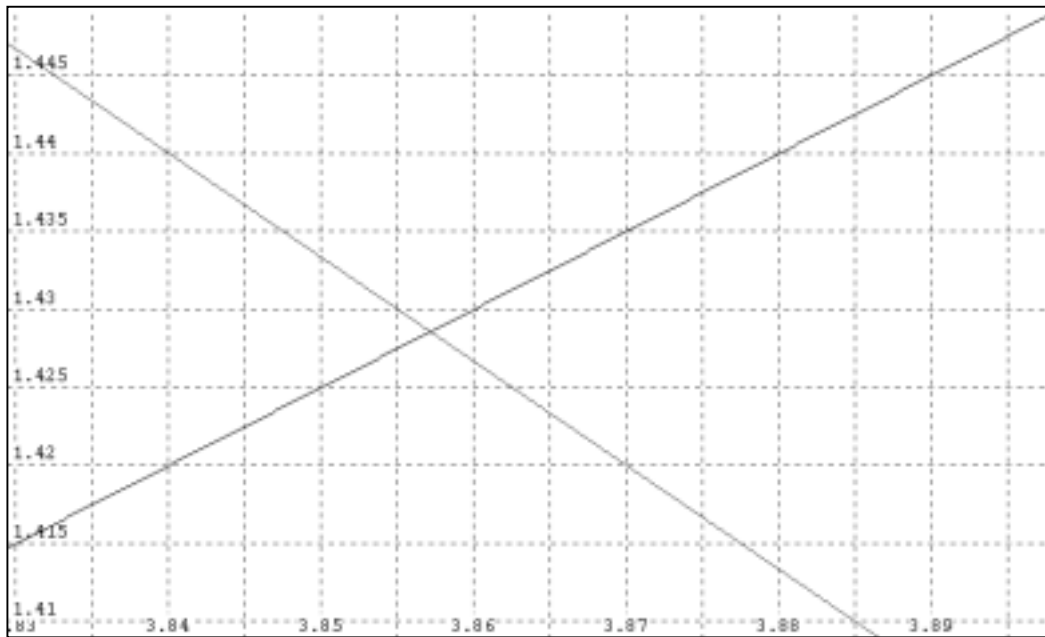
Description of the Activity:

1. The teacher gives a brief revision on the graphical method of solving simultaneous linear equations in two unknowns.
2. The teacher demonstrates how to use *Graphmatica* by using the example in Part I of the worksheet :

$$\text{Solve } \begin{cases} x - 2y = 1 \\ 2x + 3y = 12 \end{cases} \text{ graphically.}$$

The students follow the steps given by the teacher during the demonstration as a practice. The teacher should point out that *Graphmatica* does not provide accurate solutions but only provides ways to find the solutions up to a certain degree of accuracy.

3. After drawing the graphs of $x - 2y = 1$ and $2x + 3y = 12$ as shown in Figure 1 on the worksheet, the teacher asks students to answer Question 2. Students may come up with different answers. The teacher asks students to express their views. The teacher demonstrates how to use the **Zoom in** function of *Graphmatica* at a suitable level of accuracy to find the solutions in Questions 3 to 6. The figure below is a “zoom in” illustration to magnify the graph. The procedure for **Zoom in** is listed in the Operation Procedure at the end of this exemplar.



4. Students are asked to work in pairs to finish Part II of the worksheet by using *Graphmatica*.
5. After students have completed Part II, the teacher gives them the answers for checking.

Worksheet : Using Graphmatica to solve simultaneous linear equations**Part I : Example**

Use *Graphmatica* to solve the simultaneous linear equations graphically.

$$(*) \begin{cases} x - 2y = 1 \\ 2x + 3y = 12 \end{cases}$$

1. Draw the graphs of $x - 2y = 1$ and $2x + 3y = 12$ and label them as shown in Figure 1.

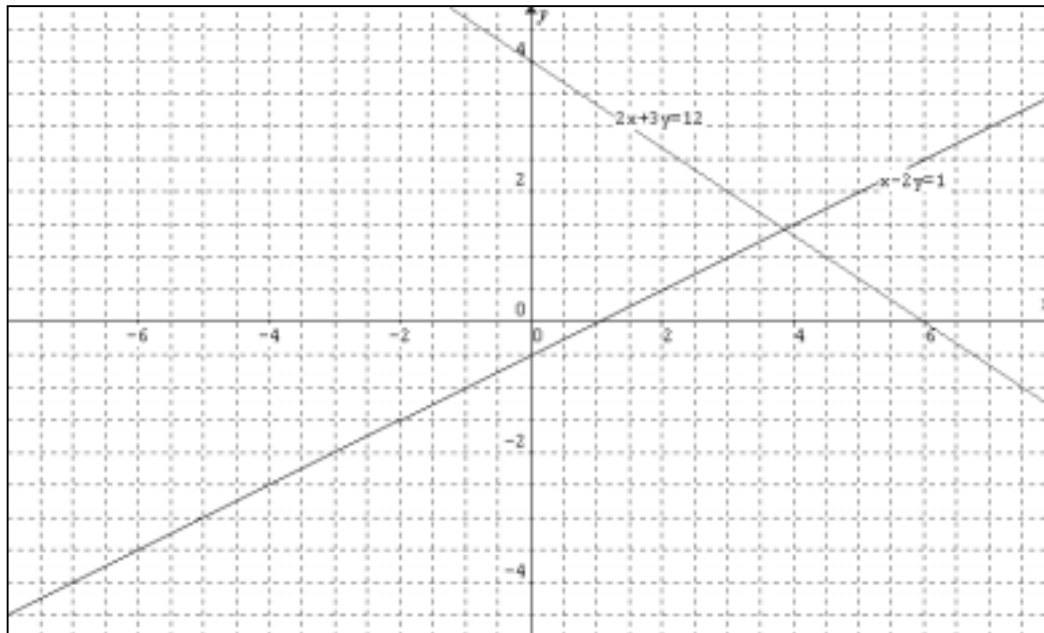


Figure 1

2. From the graph above, can you determine the solution of (*) correct to one decimal place? Why?

3. Use the **Zoom in** function to magnify the graph so that you can determine the solution of (★) correct to 1 decimal place.

From the above graph, we can see that the solution to (★) is

$$x = \underline{\hspace{2cm}}, y = \underline{\hspace{2cm}}, \text{ correct to 1 decimal place.}$$

4. Use the **Zoom in** function again to magnify the graph so that you can determine the solution of (★) correct to 2 decimal places.

From the above graph, we can see that the solution to (★) is

$$x = \underline{\hspace{2cm}}, y = \underline{\hspace{2cm}}, \text{ correct to 2 decimal places.}$$

5. Magnify the graph again so that you can determine the solution of (★) correct to 3 decimal places.

From the above graph, we can see that the solution to (★) is

$$x = \underline{\hspace{2cm}}, y = \underline{\hspace{2cm}}, \text{ correct to 3 decimal places.}$$

6. Magnify the graph once more so that you can determine the solution of (★) correct to 4 decimal places.

From the above graph, we can see that the solution to (★) is

$$x = \underline{\hspace{2cm}}, y = \underline{\hspace{2cm}}, \text{ correct to 4 decimal places.}$$

Part II : Exercise

Use *Graphmatica* to solve the following simultaneous linear equations. If the answer is not exact, correct your answer to 4 decimal places. Print out your zoomed graphs and stick them on a paper. Write down your solutions.

1.
$$\begin{cases} 2x + 5y = 6 \\ 3x - y + 6 = 0 \end{cases}$$

2.
$$\begin{cases} 2x - 3y = 12 \\ x - 5y + 5 = 0 \end{cases}$$

3.
$$\begin{cases} x - 2y - 4 = 0 \\ 5x - 3y + 10 = 0 \end{cases}$$

Notes for teachers :

1. Answers to Part I of the worksheet:

Point 2 It is not easy to determine the solution to one decimal place as both of the horizontal and vertical gridlines are not dense enough.

Point 3 The solution for (★) is $x = 3.9$, $y = 1.4$, correct to 1 decimal place.

Point 4 The solution for (★) is $x = 3.86$, $y = 1.43$, correct to 2 decimal places.

Point 5 The solution for (★) is $x = 3.857$, $y = 1.428$, correct to 3 decimal places.

Point 6 The solution for (★) is $x = 3.8571$, $y = 1.4286$, correct to 4 decimal places.

2. Answers to Part II of the worksheet:

Question	Answer
1	$x = -1.4118$, $y = 1.7647$ (correct to 4 decimal places)
2	$x = 10.7143$, $y = 3.1429$ (correct to 4 decimal places)
3	$x = -4.5714$, $y = -4.2857$ (correct to 4 decimal places)

3. For less able students, the teacher may provide some exercises on the rounding off of numbers before attempting Part II of the worksheet. Moreover, students can print out graphs of different scales for comparison.

4. When magnifying the graph, the teacher should instruct students to magnify the graph to a scale large enough to find the required number of decimal places.

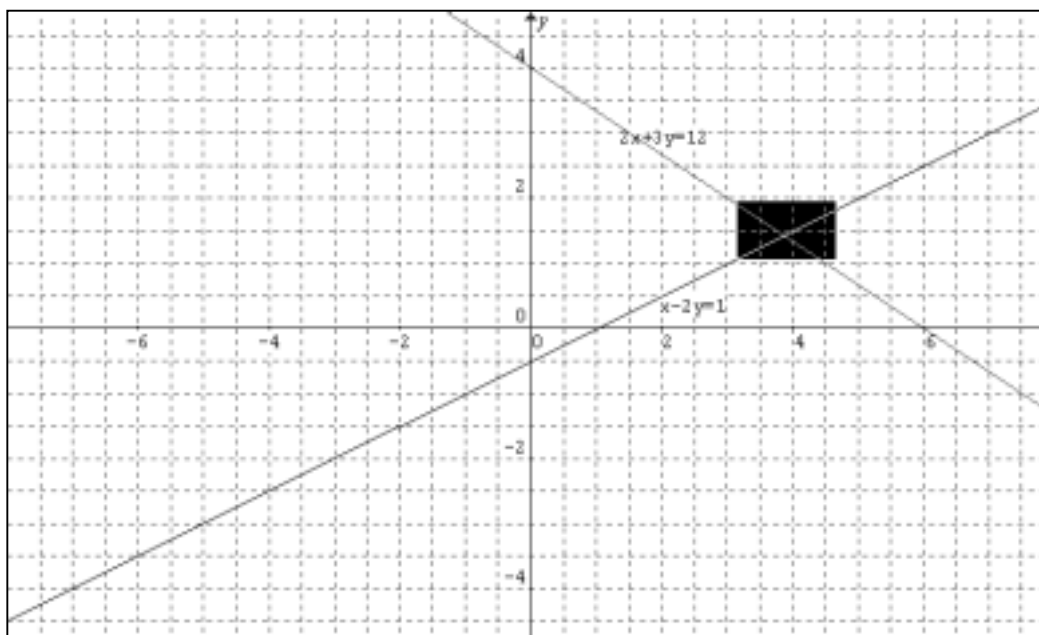
5. For more able students, they can use this worksheet as a self-learning exercise by following the Operation Procedure on their own.


Operation Procedure :

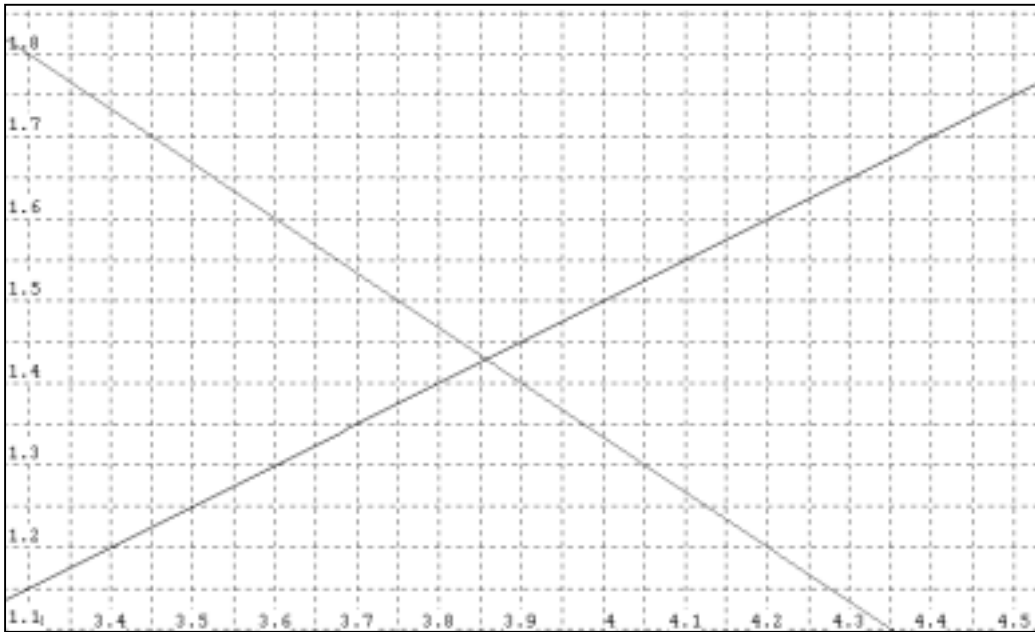
Using *Graphmatica* to solve the simultaneous linear equations


$$\begin{cases} x - 2y = 1 \\ 2x + 3y = 12 \end{cases} \text{ graphically.}$$

1. The demo version of *Graphmatica* may be downloaded from the web site <http://www8.pair.com/ksoft/>.
2. After launching the program *Graphmatica*, input the equation $x - 2y = 1$ in the dialogue box and then press **Enter**. A line will be plotted.
3. Plot the line $2x + 3y = 12$ in a similar way.
4. To change the colour of the graph, select **View | Colors**. In the dialogue box, select the colour preferred.
5. To label the graph of the equation $x - 2y = 1$, select **Labels | Annotate**. In the dialogue box, enter the equation ' $x - 2y = 1$ ' and press **Place**. Go to the desired location on the graph and click the mouse to label it. Repeat similar procedures for the graph of the equation $2x + 3y = 12$.
6. To magnify the graph, use the **Zoom in** function. Move the mouse to a point near the upper-left corner of the point of intersection of the two graphs. Drag it to select a rectangular region for zooming. See the figure below.



7. To zoom in, click the **Zoom in** icon  to obtain the result shown in the figure below.



8. To obtain the solution correct to any decimal places, zoom in until you get to the sufficient level of accuracy.
9. Click the **Default grid** icon  to go back to the default size of the graph.
10. To create a scroll bar for the graph, select **Options | Show Scrollbars**.