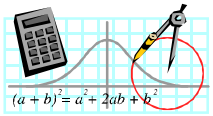

Exemplar 14: Number of Solutions of Simultaneous Linear Equations

Objective: To explore the number of solutions of simultaneous linear equations

Key Stage: 3

Learning Unit: Linear Equations in Two Unknowns

Materials Required: *Excel* and the file NA14_e.xls

Prerequisite Knowledge: (1) Graphs of linear equations in two unknowns
(2) Solving simultaneous linear equations by an algebraic method and the graphical method

Description of the Activity:

1. The teacher gives a brief revision on the methods of solving simultaneous linear equations by an algebraic method.

2. Students are divided into groups.

3. The teacher distributes the worksheet to students.

4. Students are asked to answer Question 1 in the worksheet by an algebraic method.

5. After completing Question 1, students open the *Excel* file NA14_e.xls and do Questions 2 to 7. A visual Basic Application programme is contained in the file. The programme enables students to see the interactive change on the graphs when clicking the built-in buttons (see Figure 1).

6. Some group representatives are invited to present their findings or conclusions to the class. The teacher gives comments on their conclusions.

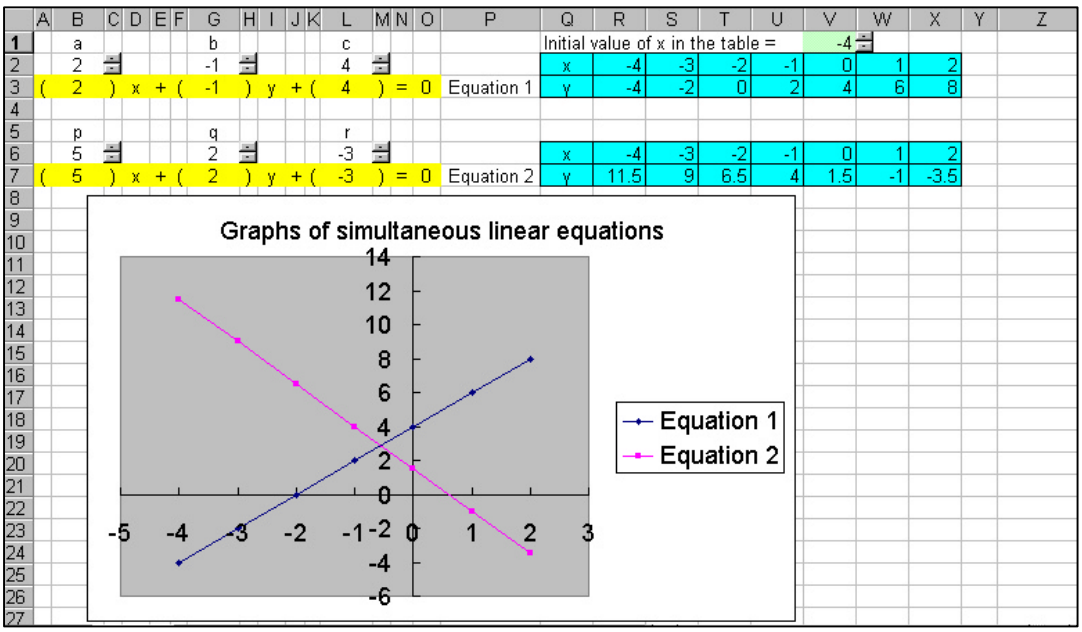
a
b
c
2
-1
4
(2) x + (-1) y + (4) = 0
Equation 1
p
q
r
5
2
-3
(5) x + (2) y + (-3) = 0
Equation 2
Initial value of x in the table = -4
x -4 -3 -2 -1 0 1 2
y -4 -2 0 2 4 6 8
x -4 -3 -2 -1 0 1 2
y 11.5 9 6.5 4 1.5 -1 -3.5
Graphs of simultaneous linear equations
Equation 1
Equation 2

Figure 1

Worksheet : Number of solutions of simultaneous linear equations

Part I

1. Solve the following simultaneous linear equations by an algebraic method:

$$\begin{cases} 3x - y = 0 \\ 2x - 3y + 7 = 0 \end{cases}$$

2. Open the *Excel* file NA14_e.xls.

The *Excel* file contains a program which is used to solve the simultaneous equations $\begin{cases} ax + by + c = 0 \\ px + qy + r = 0 \end{cases}$ graphically,

where a, b, c, p, q and r are real numbers.

Solve the simultaneous linear equations in question 1 again by putting the values of a, b, c, p, q and r in the cells B2, G2, L2, B6, G6 and L6 respectively.

You can read the solution either from the graphs or from the tables.

Answer:

(a) The number of the point(s) of intersection of the graphs is _____.

(b) The number of solution(s) is _____.

(c) The solution is (,).

3. Use the *Excel* file to find the number of solution(s) of the following simultaneous linear equations and the number of point(s) of intersection of their graphs.

(a)

Equation 1 $ax + by + c = 0$	**Equation 2** $px + qy + r = 0$	**Number of solution(s)**	**Number of point(s) of intersection**	$\frac{a}{p}$	$\frac{b}{q}$
$3x - y = 0$	$2x - 3y + 7 = 0$				
$3x - y + 1 = 0$	$2x - 3y + 7 = 0$				
$3x - y + 2 = 0$	$2x - 3y + 7 = 0$				
$3x - y + 3 = 0$	$2x - 3y + 7 = 0$				
$3x - y + 4 = 0$	$2x - 3y + 7 = 0$				
$3x - y + 5 = 0$	$2x - 3y + 7 = 0$				
$3x - y + 6 = 0$	$2x - 3y + 7 = 0$				
$3x - y + 7 = 0$	$2x - 3y + 7 = 0$				
$3x - y + 8 = 0$	$2x - 3y + 7 = 0$				

Table 1

You can change the value of c by entering your value in the cell L2 directly or clicking the **SpinUp** or **SpinDown** buttons in the cell M2.
Clicking the **SpinUp** or **SpinDown** buttons in the cells W1 can change the initial value of *x* in the tables.

(b) What happens to the graph of Equation 1 when you click the **SpinButton** in the cell M2 to change the value of c?

_____

_____

(c) From Table 1, can you see the relation between $\frac{a}{p}$ and $\frac{b}{q}$? What is the relation?

_____

_____

(d) Now choose some other values for a, b and c. Input the values in the *Excel* file. For each value of a and b, click the **SpinButton**  of c to change it to different values.

Observe the effect of different values of c on the number of solutions. Write down your conclusion below. Discuss with your group members.

_____

_____

_____

_____

Part II

4. Input values of a, b, c, p, q and r in the *Excel* file according to the simultaneous linear equations: $\begin{cases} 2x - y + 4 = 0 \\ -4x + 2y - 3 = 0 \end{cases}$.

   (a) Do the graphs of the two equations intersect? _____

   (b) How many solution(s) satisfy the equations? _____

   (c) What is the relation between these two graphs? _____

   (d) What is the relation between the number of intersecting points and the number of solutions ?

   _____

   _____

   You can alter the initial value of *x* by clicking the **SpinButton**  in the cell W1 to see whether the two graphs intersect or not.

5. In some cases, there are no points of intersection for the graphs and therefore, no solutions for the simultaneous linear equations.

   (a) Some of the equations in Table 2 are omitted. You are now required to construct suitable equations so that there are <u>no solutions</u> for the simultaneous linear equations in each of the questions given in Table 2. Check your answers by the programme provided in the Excel file. Write your answers in the table.

Question	**Equation 1** $\mathbf{ax + by + c = 0}$	**Equation 2** $\mathbf{px + qy + r = 0}$	$\mathbf{\frac{a}{p}}$	$\mathbf{\frac{b}{q}}$	$\mathbf{\frac{c}{r}}$
1	$x + 2y - 3 = 0$				
2	$3x + y + 1 = 0$				
3	$x - 2y = 0$				
4		$4x + 3y - 5 = 0$			
5		$-2x + y + 10 = 0$			
6		$x + y + 3 = 0$			

Table 2

(b) Given that the simultaneous equations $\begin{cases} ax + by + c = 0 \\ px + qy + r = 0 \end{cases}$ have no solutions, write down the relations, if any, between a, b, c, p, q and r with reference to the results in Table 2.

_____

_____

Part III

6. Given two simultaneous linear equations $\begin{cases} 2x - y + 4 = 0 \\ -4x + 2y - 3 = 0 \end{cases}$, input the corresponding values of a, b, c, p, q and r in the Excel file. Now choose –3 as the value of *r* for the second equation.

   (a) Try to change the value of r by clicking the corresponding **SpinButton** so that the two graphs coincide.

   What is the value of r? _____

   (b) Can you suggest some possible solutions from the tables in the *Excel* file? Write down some of the solutions here:
   (,), (,), (,) , (,).

   (c) From the above results, can you draw any conclusion? Write it down.

   _____

   _____

   (d) What are the relations between a, b, c, p, q and r in this case when the two graphs of the equations $ax + by + c = 0$ and $px + qy + r = 0$ coincide?

   _____

   _____

Part IV

Conclusion

7. Summarize all your findings in Parts I, II and III and write your conclusions below.

Given two equations $\begin{cases} ax + by + c = 0 \\ px + qy + r = 0 \end{cases}$, complete the following table.

Number of point(s) of intersection	**Number of solution(s)**	**Relationship between $\frac{a}{p}$, $\frac{b}{q}$ and $\frac{c}{r}$**
0		
1		
Infinitely many		

Notes for teachers:

1. The objective of this exemplar is to explore the number of solutions of simultaneous linear equations rather than to find the solution(s). The graphs are used to show points of intersection only. Part I focuses on two non-parallel lines, Part II on two parallel lines and Part III on two overlapping lines.

2. From the graphs, it is obvious that the number of points of intersection can be 0, 1 or infinitely many. Students may use "*more than one solution*" to describe the case with infinitely many solutions. The teacher may introduce the term "infinitely many solutions", "one solution" and "no solution" to students. The terms "consistent" and "inconsistent" may also be introduced to the more able students.

3. In the *Excel* file, if we set the value of b to zero in the cell G2, we will have problems in plotting the graph, because, in the table, the value of *y* is calculated from the formula $y = \frac{-(ax + c)}{b}$. *y* is only well-defined when b is not equal to zero.

   The teacher may point out that this is a special case and it represents a vertical line.

4. Answers to the worksheet:

<table>
<tr><th>Question</th><th colspan="7">Answer</th></tr>
<tr><td>1</td><td colspan="7">(1, 3)</td></tr>
<tr><td>2</td><td colspan="7">(a) 1
(b) 1
(c) (1, 3)</td></tr>
<tr><td rowspan="10">3</td><td rowspan="10">(a)</td><td>Equation 1
$ax + by + c = 0$</td><td>Equation 2
$px + qy + r = 0$</td><td>Number of solution(s)</td><td>Number of point(s) of intersection</td><td>$\frac{a}{p}$</td><td>$\frac{b}{q}$</td></tr>
<tr><td>$3x - y = 0$</td><td>$2x - 3y + 7 = 0$</td><td>1</td><td>1</td><td>$\frac{3}{2}$</td><td>$\frac{1}{3}$</td></tr>
<tr><td>$3x - y + 1 = 0$</td><td>$2x - 3y + 7 = 0$</td><td>1</td><td>1</td><td>$\frac{3}{2}$</td><td>$\frac{1}{3}$</td></tr>
<tr><td>$3x - y + 2 = 0$</td><td>$2x - 3y + 7 = 0$</td><td>1</td><td>1</td><td>$\frac{3}{2}$</td><td>$\frac{1}{3}$</td></tr>
<tr><td>$3x - y + 3 = 0$</td><td>$2x - 3y + 7 = 0$</td><td>1</td><td>1</td><td>$\frac{3}{2}$</td><td>$\frac{1}{3}$</td></tr>
<tr><td>$3x - y + 4 = 0$</td><td>$2x - 3y + 7 = 0$</td><td>1</td><td>1</td><td>$\frac{3}{2}$</td><td>$\frac{1}{3}$</td></tr>
<tr><td>$3x - y + 5 = 0$</td><td>$2x - 3y + 7 = 0$</td><td>1</td><td>1</td><td>$\frac{3}{2}$</td><td>$\frac{1}{3}$</td></tr>
<tr><td>$3x - y + 6 = 0$</td><td>$2x - 3y + 7 = 0$</td><td>1</td><td>1</td><td>$\frac{3}{2}$</td><td>$\frac{1}{3}$</td></tr>
</table>

<table>
<tr><th>Question</th><th colspan="8">Answer</th></tr>
<tr><td></td><td></td><td>$3x - y + 7 = 0$</td><td>$2x - 3y + 7 = 0$</td><td>1</td><td>1</td><td>$\frac{3}{2}$</td><td>$\frac{1}{3}$</td></tr>
<tr><td></td><td></td><td>$3x - y + 8 = 0$</td><td>$2x - 3y + 7 = 0$</td><td>1</td><td>1</td><td>$\frac{3}{2}$</td><td>$\frac{1}{3}$</td></tr>
<tr><td></td><td colspan="8">(b) The graph moves upwards or downwards and is parallel to the original line.
(c) $\frac{a}{p}$ is not equal to $\frac{b}{q}$.
(d) The change in the value of c will have no effects on the number of solution(s) if the original equations have only one solution.</td></tr>
<tr><td>4</td><td colspan="8">(a) No
(b) 0
(c) Parallel to each other
(d) The same</td></tr>
<tr><td>5</td><td>(a)</td><td>Question</td><td>Equation 1
$ax + by + c = 0$</td><td>Equation 2
$px + qy + r = 0$</td><td>$\frac{a}{p}$</td><td>$\frac{b}{q}$</td><td>$\frac{c}{r}$</td></tr>
<tr><td></td><td></td><td>1</td><td>$x + 2y - 3 = 0$</td><td>$\boldsymbol{x + 2y = 0}$</td><td>1</td><td>1</td><td>$-\frac{3}{2}$</td></tr>
<tr><td></td><td></td><td>2</td><td>$3x + y + 1 = 0$</td><td>$\boldsymbol{6x + 2y + 3 = 0}$</td><td>$\frac{1}{2}$</td><td>$\frac{1}{2}$</td><td>$\frac{1}{3}$</td></tr>
<tr><td></td><td></td><td>3</td><td>$x - 2y = 0$</td><td>$\boldsymbol{x - 2y + 5 = 0}$</td><td>1</td><td>1</td><td>0</td></tr>
<tr><td></td><td></td><td>4</td><td>$\boldsymbol{4x + 3y + 2 = 0}$</td><td>$4x + 3y - 5 = 0$</td><td>1</td><td>1</td><td>$-\frac{2}{5}$</td></tr>
<tr><td></td><td></td><td>5</td><td>$\boldsymbol{2x - y + 10 = 0}$</td><td>$-2x + y + 10 = 0$</td><td>–1</td><td>–1</td><td>1</td></tr>
<tr><td></td><td></td><td>6</td><td>$\boldsymbol{-3x - 3y + 1 = 0}$</td><td>$x + y + 3 = 0$</td><td>–3</td><td>–3</td><td>$\frac{1}{3}$</td></tr>
<tr><td></td><td colspan="8">(b) $\frac{a}{p} = \frac{b}{q} \neq \frac{c}{r}$</td></tr>
<tr><td>6</td><td colspan="8">(a) –8
(b) (–2,0), (–1,2), (0,4), (1,6), etc.
(c) When two lines overlap, there are infinitely many solutions.
(d) $\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$</td></tr>
</table>

Question	Answer		
7	Number of point(s) of intersection	Number of solution(s)	Relationship between $\frac{a}{p}$, $\frac{b}{q}$ and $\frac{c}{r}$
	0	0	$\frac{a}{p} = \frac{b}{q} \neq \frac{c}{r}$
	1	1	$\frac{a}{p} \neq \frac{b}{q}$
	Infinitely many	Infinitely many	$\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$

Note The answers to 5(a) in bold are not unique. There might be other equations satisfying the given condition.