B. Learning Objectives and Notes on Teaching (KS3)

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<tr><th>Unit</th><th>Learning objectives</th><th>Notes on teaching</th><th>Time ratio</th></tr>
<tr><td colspan="4">Number and Number Systems</td></tr>
<tr><td>Directed Numbers and the Number Line</td><td>• understand and accept intuitively the concept and uses of negative numbers
• have simple ideas of ordering on the number line
• explore and discuss the manipulation of directed numbers
• manipulate directed numbers</td><td rowspan="2">In primary school levels, students have understood the concepts of different forms of numbers and learnt to interconvert numbers in different forms (refer to Annex I of the Syllabus for further details). Teachers should note that students studied the TOC programme in their primary schools are not expected to have any ideas of negative numbers whereas students studied the non-TOC programme may have the intuitive ideas on the opposite meanings of negative and positive numbers. In secondary schools, students will extend the concepts of numbers from positive numbers to directed numbers and then to real numbers. However, teachers should assess and provide consolidation activities, whenever necessary, to ensure that students have firm foundations on the concepts of numbers before proceeding to the study in KS3 and KS4. Students will encounter rational and irrational numbers in KS3 as a natural consequence of introducing “Pythagoras’ Theorem” or “Trigonometric Ratios and Using Trigonometry”. Teachers could arrange the unit “Rational and Irrational Numbers” together with any one of the above two units. Manipulations of surds are confined to techniques sufficient for handling problems related to the above 2 units. The formal introduction to the real number systems will be studied in KS4.</td><td>12</td></tr>
<tr><td>Rational and Irrational Numbers</td><td>• be aware of the existence of irrational numbers and surds
• explore the representations of these numbers in the number line
• <u>manipulate commonly encountered surds including the rationalization of the denominator in the form of $\sqrt{a}$</u>
• <u>appreciate the expressions of surds could be expressed in a more concise form</u></td><td>6</td></tr>
<tr><td>Numerical Estimation</td><td>• be aware of the need to use estimation strategies in real-life situations and appreciate the past attempts to approximate values such as π in the past
• determine whether to estimate values or to compute the exact values
• select and use estimation strategies to estimate values and to judge the reasonableness of results
• choose appropriate means for calculations such as mental computation, calculators or paper and pencil etc.</td><td>Some students are uncomfortable with estimation, as most of their mathematics exercises, especially in primary schools, require them to give the exact values as answers. In the secondary school level, the teacher should help students build up flexible thinking in computations. Through the activities in the unit “Numerical Estimation”, students are expected to be aware of the use of estimation in various activities and be able to apply simple estimation strategies to estimate values and to check the reasonableness of results obtained. Students need to be convinced that estimation is sometimes necessary. Hence, teachers are recommended to use simple examples in the early stages and avoid requiring very high precision in estimates throughout. Teachers can emphasis the use of the language of estimation, such as</td><td>5</td></tr>
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		◆ about 25, ◆ close to 25, ◆ a little less than 25, ◆ between 24 and 25, but probably a little bit closer to 25, ◆ somewhere between 24 and 25, etc. In order to help students make estimation a part of their everyday experiences, teachers should use real-world applications extensively. Estimation, like problem solving skill, calls on a variety of skills. It is developed and improved over a long period of time as it involves an attitude as well as a set of skills. Estimation must be emphasized regularly throughout all mathematics topics. Estimation can be emphasized through exercises on daily assignments and in lessons by asking students to response orally on the estimated values.	
Approximation and Errors	● acquire further concepts and skills of rounding off numbers to a required number of significant figures ● understand the meaning of scientific notation ● use scientific notation in practical problems ● be aware of the size of errors during estimation and approximation understand and calculate different types of errors such as absolute errors, relative errors and percentage errors	For daily life activities, students should sense when it is appropriate to estimate and recognize how close, or precise, an estimate is required in a given situation. For the unit "Approximation and Errors", the teacher can extend the ideas of rounding off to the number of significant figures and the degree of accuracy required in expressing a number in the scientific notation. Students are expected to be fluent in approximating a given number to the required degree of accuracy. Nevertheless, it is very important that students can determine the degree of accuracy in a given context and be sensible in the relative size of errors in the estimation or the approximation process.	7

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Comparing Quantities			
Using Percentages	• understand the meaning of percentages and percentage changes • apply percentage changes to solve simple buying and selling problems • apply percentages to solve problems involving simple and compound interests, growth and depreciation.	In the primary level, students have learnt the meaning of percentage, that is, a fraction with 100 as the denominator and the percentage is the most convenient way in comparing fractions. They have learnt to interconvert between percentages and decimals, percentage and fractions. They have also learnt to apply percentages to solve simple daily-life problems such as discount and simple interest (but not for those students who studied the TOC programme). At this stage, students are expected to use algebraic symbols and the ideas of percentages to compare quantities and to solve more real-life problems. In the unit "Using Percentages", only simple problems are expected. The comparison of more than 2 quantities is expected to be learnt in the unit "More about percentages". By the end of the nine-year basic education, students are expected to have basic understanding on the applications of percentages in the real-life contexts, including taxation and rates. More complicated contextual problems will be introduced in the unit "Further Applications" in KS4.	17
More about Percentages	• apply percentages to solve further practical problems involving successive and component changes • apply percentages to solve simple real-life problems involving taxation and rates		7
Rate and Ratio	• understand the meaning of rate and ratio • recognize the notation of a : b, a : b : c • apply the ability in using rate, ratio to solve real-life problems including mensuration problems	In primary levels, students have learnt the idea of ratio through manipulations of fractions and the idea of rate in solving problems related to speed. They also have simple ideas of direct and inverse proportions and are able to solve simple related problems using the unitary method. They should learn that a ratio is unaltered if the two numbers (or quantities) of the ratio are both multiplied or divided by the same number. In KS3, students are expected to understand the difference between the meaning of rate and ratio. They should learn the notation of a two-term ratio as a:b, and to extend it to a three-term ratio as a:b:c. Students should be able to deal with rate and ratio in examples from mensuration including similar triangles, examples in sciences or Geography (on R.F.). The formal treatment on the direct and inverse proportions from the perspective of algebraic relations between quantities is not expected until KS4.	8

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Observing Patterns and Expressing Generality			
Formulating Problems with Algebraic Language	• appreciate the use of letters to represent numbers • understand the language of algebra including translating word phrases into algebraic expressions or write descriptive statements for algebraic expressions • note the differences between the language of arithmetic and the language of algebra • recognize some common and simple formulas which can be expressed as algebraic forms and be able to substitute values • formulate simple algebraic equations/ inequalities to solve problems • investigate, appreciate and observe the patterns of various number sequences such as polygonal numbers, arithmetic and geometric sequences, Fibonacci sequence etc. • use algebraic symbols to represent the number patterns • obtain a preliminary idea of function such as the input-processing-output concept	Algebraic techniques are important for students to learn many topics in mathematics. Past studies suggested that many students failed to comprehend elementary algebra because of an inadequate understanding of arithmetic. Sufficient consolidation activities should be provided if students cannot master arithmetic properly. The smooth transition from number to algebra is deemed crucial and students should be guided to see the difference between the two languages. In this module, teachers are expected to introduce the algebraic language to students through activities of observing patterns, generalizing, modeling and justifying. These processes are essential for beginners in algebra. In the module “Algebraic Relations and Functions”, students are expected to manipulate different types of algebraic relations. The unit “Formulating Problems with Algebraic Language” is considered an introduction to the algebraic language with letters to represent unknowns and variables. The unit “Manipulation of Simple Polynomials” is intended to provide students with activities to manipulate simple polynomials involving more than one variable. Factorization of simple polynomials is considered as reverse process of expansion or multiplication. After these exploratory activities, students will further explore and generalize algebraic rules relating to integral indices in the unit “Laws of Integral Indices”. Although students have engaged in finding the successive terms of a sequence (not necessarily confined to numbers) in primary levels, students at this stage are expected to use algebraic symbols in generalizing the patterns. Activities on sequence in the unit “Formulating Problems with Algebraic Language”, students get involved in the process of observing patterns of numbers, making conjectures on the patterns and describing them with the algebraic language. Through generating terms from a symbolic expression of the general term of a sequence, students will obtain a preliminary idea of the input-processing-output concept and recognize the different meaning of algebraic symbols such as $2x$ and x^2. Arithmetic sequence and geometric sequence are introduced as examples to consolidate their understanding on number sequences. Introducing the formula on finding the general terms should be deferred to KS4 . Teachers can make use of spreadsheets or calculators to relief students from the burden of calculations. These IT tools can also allow a free, two-way movement between the world of numbers and that of algebra. By formulating	14

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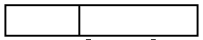
		and solving simple problems with the algebraic language, students realize the advantage of using the algebraic method over the traditional arithmetic method. It is important for students to recognize the conventions for writing simple algebraic expressions and the difference between algebraic and arithmetic languages. For example, ab = a × b whereas 57 = 5 tens + 7.	
Manipulations of Simple Polynomials	• recognize polynomial as a special example of algebraic expressions • recognize the meaning of the terminology involved • add, subtract, multiply polynomials involving more than one variable	Terminology such as degree, terms and coefficients is introduced in the unit “Manipulations of Simple Polynomials”. Activities for students to identify like terms, unlike terms and coefficients should be provided so as to facilitate the discussion on their manipulations. The addition and subtraction of like terms are expected. The distributive property of polynomials can be introduced with blocks to demonstrate the equivalence.  Teachers can relate the numeric distributive property to introduce the case for polynomials, such as a(b + c) = ab + ac analogy to 2(99 + 1) = 2 × 99 + 2 × 1. Thorough discussions on how to handle brackets should be provided to enable students to distinguish cases where distributive property applies. In this unit, students will have intuitive ideas on the rules of exponents such as $x^2\ x^3 = x^5$. Detailed discussions on the laws of indices will be introduced in the unit “Laws of Integral Indices”.	10
Laws of Integral Indices	• extend and explore the meaning of the index notation of numbers with negative exponents • explore, understand and use the laws of integral indices to simplify simple algebraic expressions (up to 2 variables only) • <u>understand and compare numbers expressed in various bases in real-life situations</u> • <u>foster a sense of place values in different numeral systems</u> • <u>inter-convert between simple binary/hexadecimal numbers to decimal numbers</u>	Students have the idea of positive exponents for numbers in the primary level. In the unit “Laws of Integral Indices”, students will study the meaning of negative exponents. Activities to explore rules on exponents for numbers should be provided prior to the introduction of the laws of integral indices. Students will not have any problems in handling algebraic rules when they are handled individually. However, when the use of a few rules at the same time was expected, past experiences revealed that many students had difficulties in understanding the rules involved and many “careless mistakes” occurred. For example, ◆ confusions on the distributive rule such as $m^3n^2 = (mn)^5$, $(a+b)^2=a^2 + b^2$ ◆ unable to recognize the sequence of operations such as $3 \times 4^6 = 12^6$, $6^4 / 3^2= 2^2$ ◆ confusions in addition or multiplication, such as $(x^3)^2 = x^5$, $3a{\cdot}2a = 5a$ ◆ operating on numbers and neglecting the letters such as $4 + 3n = 7n$, $3{\cdot}2^x = 6^x$ Clarifying students’ misconceptions are important for further exploration in	10

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		the topic. Nevertheless, teachers should not ask students to go into tedious manipulations of algebraic symbols. As a base line, teachers should make sure that students possess the necessary abilities in tackling equations and inequalities. Numbers expressed in various bases in real-life situations are considered as an application of exponents. Simple inter-conversion between binary/hexadecimal numbers and decimal numbers is to foster a strong sense of place values. The conversion can be done as follows: $35 = 32 + 2 + 1 = 2^5 + 2^1 + 2^0 = 100000_2 + 10_2 + 1_2 = 100011_2$ The scientific notation in the unit "Approximation and Errors" can also be seen as another application of the laws of integral indices.	
Factorization of Simple Polynomials	• understand factorization as a reverse process of expansion • factorize polynomials by using common factors and grouping of terms • factorize polynomials by using identities including difference of two squares; perfect square expressions; difference and sum of two cubes • factorize polynomials by cross-method	Factorization of simple polynomials is introduced as a reverse process of expansion. Students can countercheck the factorization result with the expansion (multiplication) technique. As students are expected to apply some identities in factorizing polynomials, teachers should ensure students to have learnt the unit "Identities" prior to this unit.	15

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Algebraic Relations and Functions			
Linear Equations in One Unknown	• formulate and solve linear equations in one unknown • **solve literal equations	Students in primary levels have learnt to solve simple linear equations with the solution up to 2 steps only. For the unit "Formulating Problems with Algebraic Language", students have further ideas on problems expressing in equations and inequalities. In that unit, they are expected to solve the equations or inequalities by "guess and check" or by direct inspection. In this module, students are expected to explore different types of algebraic relations that can be described in equations, formulas, identities or inequalities. They will explore the meanings of these different types of algebraic relations further and learn the formal method to solve equations in this unit and inequalities in the Unit "Linear Inequality in One Unknown". Students are not expected to learn the terminology "open sentence". However, they should have the idea of substituting values to have the relation becoming true and these values are called solutions for the equation. Solutions in these different relations should be contrasted and thoroughly discussed. For example, teachers can arrange the unit "Linear Equations in One Unknown" together with the unit "Linear Inequalities in One Unknown" to contrast the difference in their solutions. Students have learnt to solve linear equations in one unknown using the idea of a balance in their primary levels. In the beginning of KS3, students are still expected to apply the same idea to understand the steps in solving linear equations in one unknown. When they are familiar with the idea, they can apply the rules of transposing terms in solving equations. Terms such as "solution" or "root" can be introduced. Past experiences revealed that students always mixed up expressions with equations. Teachers should alert students their differences.	7
Linear Equations in Two Unknowns	• plot and explore the graphs of linear equations in 2 unknowns • formulate and solve simultaneous equations by algebraic and graphical methods • be aware of the approximate nature of the graphical method • **explore simultaneous equations that are inconsistent or that have no unique solution	Before introducing the methods in solving simultaneous linear equations in two unknowns, it is important for students to have some experiences in exploring the inputs and outputs of a linear polynomial in two unknowns. Students can make use of some spreadsheet software to explore the meaning of linear equations in two unknowns. After then, teachers can introduce the graphs for linear equations in 2 unknowns. Using functions such as *zoom* and *trace* in graphical software tools can further facilitate the discussion on values not satisfying the equations. Students can explore the shapes of the graph of this type of equations with these IT tools. When students have basic understanding	15

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		on the meaning of the equation, they can proceed to solve simultaneous equations. For the algebraic method, students should have some basic ideas in changing the subject of a formula learnt in the unit "Formulas" in order to use the method of substitution to solve the equations. It is also not difficult for students to observe the meaning of solutions for simultaneous equations by the graphical approach. Abler students can also explore equations that are inconsistent or that have no unique solutions with or without the graphical tools.	
Identities	• explore the meaning of identities and to distinguish between equations and identities • discover and use the identities : difference of two squares; the perfect square expression; <u>difference and sum of two cubes</u>	It is important for students to see the difference between equations and identities. The equation is only satisfied by a certain value of x whereas the identity is satisfied by all values of x. Students can explore the meaning of identities through various approaches, such as using spreadsheet to demonstrate the substitution of values to both sides of identities; comparing the numerical or spatial patterns of the identities, etc. Students can also construct some identities of their own before they start to discover the following identities: $(x-y)(x+y) \equiv x^2 - y^2$; $(x \pm y)^2 \equiv x^2 \pm 2xy+y^2$; $(x \pm y)(x^2 \mp xy+y^2) \equiv x^3 \pm y^3$	8
Formulas	• manipulate algebraic fractions with linear factors as denominators • develop an intuitive idea of factorization of polynomials • explore familiar formulas and substitute values of formulas • perform change of subject in simple formulas but not including radical sign	Regarding the unit "Formulas", it is important for students to understand the meaning of formulas and to be able to substitute values into the formulas. Students should have the idea of change of subject in simple formulas. Simple manipulative practices are expected whereas complicated formulas or tedious steps involved can be handled with calculators or some algebraic software. If students have not got the prerequisite in the factorization of simple algebraic expressions, teachers can introduce the simple ideas of factorization such as taking out common factors or by grouping terms. However, the treatment should be simple enough for handling the requirements in the unit. Further explanation on factorization should be left to the unit "Factorization of Simple Polynomials".	14

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Linear Inequalities in One Unknown	• understand the meaning of inequality signs $\geq$, $>$, $\leq$ and $<$ • explore the fundamental properties and some laws of inequalities • solve simple linear inequalities in one unknown and represent the solution on the number line	Students should have the experience of investigating informally the meaning of inequalities such as 3x is less than 3 in the unit "Formulating Problems with Algebraic Language" by listing out values of x satisfying the statement. In this unit, students are expected to use the formal method to find the solution. Before the transposing method is introduced, students should engage in exploratory activities on the laws of inequalities, such as "If $x > y$, then $x + c > y + c$ for all values of c". Further explorations on the validity of statements, such as "If $a > b$, then $\frac{1}{a} < \frac{1}{b}$" can also be carried out so as to develop students' numbers sense. Teachers can ask students to compare the method in solving equations with that in solving inequalities, especially for the case involving negative coefficients. Contrast in solutions expressed in different inequality signs should be made, for example, the difference in the solution for $x > 3$ and $x \geq 3$. Students are also expected to represent the solution on the number line.	7