C. Learning Objectives and Teaching Suggestions (KS 4)

Unit	Learning objectives	Notes on teaching	Time ratio
Observing Patterns and Expressing Generality			
More about Polynomials	• manipulate polynomials including long division up to simple quadratic divisor • introduce the concept of division algorithm • understand and use remainder and factor theorems to factorize polynomials up to degree 3 • appreciate the power of factor theorem and also be aware of the limitation of the theorem	In this module, students are expected to further apply their algebraic skills in manipulating polynomials. Apart from addition, subtraction and multiplication of polynomials learnt in KS3, students at this stage are expected to divide polynomials by long division up to simple quadratic divisor. Simple divisions are expected to be manipulated manually by students. Exploratory work on the relation between the degree of the divisor and that of the remainder is expected. With the help of an algebraic software, the remainders and quotients from various dividends and divisors can be generated within a short time. Patterns on their degrees and their relations can be observed. Teachers can then introduce the division algorithm and the remainder theorem. The factor theorem is considered as a special case of the remainder theorem. Students are expected to apply the factor theorem to factorize polynomials. Teachers should guide students to appreciate the power of the factor theorem in finding factors. The contrast of the cross method and the factor theorem can be made. The limitations of the factor theorem in finding only linear factors should also be discussed.	9
Arithmetic and Geometric Sequences and their Summation	• explore further the properties of arithmetic and geometric sequences • develop and use the general term of the sequences • investigate and use the general formulas of the sum to n terms of arithmetic and geometric sequences • develop an intuitive idea on limit and deduce the formula for sum to infinity for certain geometric series • solve real-life problems such as interest, growth and depreciation, geometric problems etc. • ** explore recurrence relationships in some sequences	Regarding number patterns, students in KS3 make conjectures on patterns of various simple number sequences. At this stage, students are expected to study in depth 2 special examples of number sequences: arithmetic and geometric sequences. Students are expected to generalize their general terms and the formulas of the sum to n terms with simple proofs. Stories of Gauss can be introduced to motivate students in finding the rule for the summation. In deducing the formulas on the sum to infinity for the geometric series with $\lvert r \rvert < 1$, students can start with exploratory activities on values of r^n with the help of calculators or spreadsheet software. Different values of r can be compared and explored. The applications of these formulas in solving real-life problems, such as compound interest, geometric problems should be discussed. For abler students, teachers can introduce other types of sequences such as recurrence sequences.	10

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Quadratic Equations in One Unknown	• formulate and solve quadratic equations by factor method and formula • solve the equation $ax^2 + bx + c = 0$ by plotting the graph $y = ax^2 + bx + c$ and reading the x-intercept • be aware of the approximate nature of the graphical method • choose the most appropriate strategy to solve quadratic equations • recognize the conditions for the nature of roots • understand the hierarchy of real-number system and be aware of the characteristics of rational numbers when expressed in decimals (Note: Further exploration on properties of quadratic graphs would be in the unit "Function and Graphs")	At KS3, students have learnt the meanings of different types of algebraic relations. At KS4, students are expected to extend their understanding to more complicated algebraic relations. They are expected to formulate and solve quadratic and higher degree equations by both the algebraic and graphical methods. For inequalities, students will extend their exposure from linear inequalities in one unknown to two unknowns. Given two set of quantities, students are expected to explore and describe their algebraic relation, if exists. With the previous experience in exploring the input-processing-output concepts, students will be guided to interpret the basic idea of function. Different types of functions and their graphs will be explored. Quadratic, exponential and logarithmic functions will be studied in-depth to compare the properties of different types of functions. It is important that students should be guided to appreciate the use of algebraic methods in solving real-life problems. Students would be guided to factorize the expression $ax^2 + bx + c$ and then to obtain the solutions of the quadratic equation $ax^2 + bx + c = 0$. Teachers use the method of completing the square to deduce the formula for the solution of the quadratic equation. Once students understand the formula, they can use calculators to find the roots. The terminology of rational and irrational numbers should be revisited and students should have in-depth understanding on the characteristic of rational numbers. The hierarchy of the real number system should be introduced. For higher degree equations, students should use the factor theorem to solve the equation. Equations, which cannot be solved by the factor theorem, should also be discussed. The power and the limitations of graphical methods in solving equations should be explored and discussed. Graph-plotting software or graphing calculators can be used to plot the graphs of $y = ax^2 + bx + c$. The zoom functions in these tools can facilitate their discussions in tracing the intersecting points with the graph and the x-axis. Students will easily recognize the approximate nature of the graphical methods and the need to refine the solution with the help of these tools. With these IT tools, tedious algebraic manipulations or graph plotting can be minimized. More discussions should be on how to choose the most appropriate strategy in solving problems and how to formulate the problems with the algebraic language.	17
More about Equations	• formulate and solve equations which can be transformed into quadratic equations • formulate and solve one linear and one quadratic simultaneous equations by algebraic method • solve equations by reading intersecting points of given graphs • appreciate the power and understand the limitation of graphical method in solving equations • choose the most appropriate strategy to solve equations • **explore the algebraic method to solve cubic or higher degree equation		15

Unit	Learning objectives	Notes on teaching	Time ratio
Variations	• discuss the relations between 2 changing quantities • sketch the graphs of direct and inverse variations and recognize the algebraic representations between the quantities • recognize and appreciate the algebraic representations of various variations such as those in the forms of $V = \pi r^2 h$ or $y = k_1 + k_2 x$, etc. • apply the relations to solve real-life problems	In discussing the relation between quantities, students are expected to observe patterns and make generalizations on the patterns in the algebraic language. Direct and inverse variations are two fundamental and common variations in daily life. They are expected to describe these relations in both graphical and algebraic representations. The ambiguity of stating "when x is getting larger, then y is getting larger" for direct variation can be investigated. Different graphs representing the statement can be presented to facilitate the discussion. Quantities, which can be described in direct or inverse variations after transformation, should also be included. With a thorough discussion on direct and inverse variations, students can extend to represent the relations in combinations of direct and inverse variations. Real-life problems that can be modelled by these relations should be investigated.	13
Linear Inequalities in Two Unknowns	• represent the linear inequalities in 2 unknowns on a plane • discuss the solution of compound linear inequalities connected by 'and' • solve systems of linear inequalities in two unknowns • solve linear programming problems	Students at KS3 have learnt to represent the solution of linear inequalities on a number line. In the unit "Linear Inequalities in Two Unknowns, students should be guided to extend the necessity of representing solutions of linear inequalities in 2 unknowns on a plane. Students can explore the solution of the inequalities such as "$x + y > 2$" by substituting values. With the facilities in some graphing calculators or graph plotting software, students can put the tabular representations and the graphical representations side by side to facilitate discussion. With these tools, explorations on the effect of the change of inequality signs on the solution can also be done. With these basic understandings on inequalities and the prerequisite understanding on simultaneous equations, students should proceed to discuss the solution of simultaneous inequalities. The applications of linear programming in modeling real-life problems can be discussed. With the IT tools to ease students' work in graphing the feasible solution, focus can be on the strategy to set up the constraints, the objective function and the methods to find the optimum value(s) in the feasible solution. More varieties of real-life problems can be included for discussion.	15

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Functions and Graphs	• relate the idea of input-processing-output to the meaning of dependent and independent variables • understand the basic idea of a function from the tabular, symbolic and graphical representations of a function and the dummy nature of x • use the notation for a function • explore various properties of quadratic functions such as vertex, axis of symmetry, the optimum value(s) from their graphs • appreciate the contribution of Arabians on the method of completing the square and use it to find the properties of quadratic functions • appreciate the power of the method in generating a perfect square expression • sketch and compare graphs of various types of functions • solve $f(x) > k$, $f(x) < k$, $f(x) \geq k$, $f(x) \leq k$ by reading graphs of $f(x)$ • explore the effects on transformation on the functions from tabular, symbolic and graphical perspectives • visualize the effect of transformation on the graphs of functions when giving symbolic relations	At KS3, students should have the idea on input-processing-output in manipulating formulas, sequences, polynomials, etc. At this stage, the idea of dependent variables, independent variables and the intuitive idea of a function should be introduced. However, the formal definition of a function and terminology such as mapping, domain, etc. should not be included. Students have a preliminary picture on the shape of the quadratic function in the unit “Quadratic Equations in One Unknown”. In the unit “Functions and Graphs”, students should generalize the properties of quadratic functions by exploring the graphs of various quadratic functions. The algebraic method in finding the axis of symmetry and the vertex of a quadratic function is introduced. Having had the experience in quadratic functions, students can then extend their discussions to other types of functions. Teachers can use spreadsheet software or graphing calculators to investigate various functions in numeric, algebraic and graphical representations. The effect on the change in the values of the dependent variables by modifying the values of independent variables can be readily visualized with the help of these IT tools. Discussions on their shapes, growth rates, intercepts and the effects of transformations on the graphs of the functions can enhance students to have a better understanding of the algebraic symbols. Graphs of different functions can also be readily compared as they can be presented simultaneously and immediately within one screen.	16

Unit	Learning objectives	Notes on teaching	Time ratio
Exponential and Logarithmic Functions	• understand and use the laws of rational indices • understand the definition of logarithmic functions and recognize the common logarithm is not the only type of the function • examine the properties of the graphs of exponential and logarithmic functions • explore and study the relations between the properties of logarithmic function and that of exponential function • appreciate the application of logarithm in various real-life problems	In the unit “Exponential and Logarithmic Functions”, students are expected to extend the laws of indices from integral exponents to rational exponents. Manipulations on simple expressions are required so as to enable students to understand the meaning of rational indices, to clarify mistakes, such as $x^{\frac{1}{3}} = \frac{1}{x^3}$, etc. Concepts on exponents are further elaborated in introducing exponential and logarithmic functions. The relation between these two functions should be introduced so as to enable students to develop an idea on the inverse function and have a better understanding on the nature of logarithmic function. Nevertheless, the symbolic notation and the formal definition on the inverse function should not be included. Their inter-related properties can be studied from comparing their graphs. The applications of the logarithmic function in various disciplines such as dB for sound intensity, Richter scale for identifying the magnitude of an earthquake, a technique for transforming data from exponential form to linear form, etc. may be discussed. Past attempts in applying logarithm in tackling large numbers can be mentioned but should not be emphasised. (Note: The definition of logarithmic functions should not be confined on the common logarithm.)	18