Exemplar 1 :
To Formulate Algebraic Equations In Word Problems

Objectives : Students will be able to
(1) formulate an equation to solve the word problem;
(2) understand different problem solving strategies.

Dimension : Number and Algebra

Learning Unit : Formulating Problems with Algebraic Language

Key Stage : 3

Materials Required : Calculators

Prerequisite Knowledge :
(1) Using letters to represent numbers
(2) Translating word phases into algebraic expressions
(3) Solving simple linear equations

Main HOTS Involved : Problem Solving Skills, Communicating Skills

Description of the Activity :

1. Distribute Worksheet 1.1 to students and ask them to work independently for parts (a) and (b).

2. For parts (c) and (d), the teacher helps students divide into groups. Discussions among students are encouraged. The teacher asks students to suggest methods to solve the problem. Comparison of these methods can be made. Sufficient time for discussion should be allowed.

3. The following questions are asked to guide students to solve each problem in the worksheet:
   (a) What is/are the unknown(s)?
   (b) What is/are given?
   (c) Is it possible to solve the problem with the given information?
   (d) Can you obtain the solution by inspection?
   (e) Can you suggest a/some letter(s) for the unknown(s)?

(f) What strategy will you use to solve the problem?

(g) Which method is the best to solve the problem?

(h) Is the solution reasonable with reference to the real situation?

(i) Is there any alternative approach to solve the problem?

4. If students set up an equation for the problem, ask them to solve it. A verification of the solution is necessary.

Worksheet 1.1 Solving Word Problems

Edmund buys some soft drinks and fried chicken wings for a tea party. Soft drinks cost $5 per can and chicken wings cost $8 each.

(a) Eight people including Edmund join the party and each one has 1 can of soft drink and 2 chicken wings. How much does Edmund pay to prepare for the party?

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(b) Nine people including Edmund join the party and each one has 3 cans of soft drinks and a certain number of chicken wings. Edmund has to pay $423. How many chicken wings does each one have?

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(c) There are n people including Edmund joining the party and each one has x cans of soft drinks and $2x$ chicken wings. Edmund pays $273.

(i) How many people are there?

_____

_____

(ii) How many cans of soft drinks and chicken wings does each one have?

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_____

(For abler students only)

(d) A group of people including Edmund go to the party and each one has the same number of soft drinks and same number of chicken wings as the others. Edmund pays $378.

(i) How many people are there?

_____

_____

_____

(ii) How many cans of soft drinks and chicken wings does each one have?

_____

_____

_____

_____

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Notes for Teachers :

1. In this activity, students are expected to develop problem solving skills through discussion during heuristic learning processes, applying mathematical ideas and skills to unfamiliar questions, and evaluating or justifying the solution. The possible strategies (not exhaustive) applied in each question are summarised in the following table.

Problem	Possible Strategies
(a)	Using arithmetic method
(b)	Using arithmetic method / Setting up an equation
(c)	Trial and error / Setting up an equation
(d)	Trial and error / Setting up an equation & tabulating feasible solutions

2. Problem (a) is very simple. Students can just use the basic arithmetic operations to get the answer.

3. Students may solve Problem (b) by simplifying the arithmetic expression $\left(\frac{\frac{423}{9}-5\times 3}{8}\right)$ or by setting up a simple equation ($9(5\times 3+8x) = 423$). The teacher can ask students to compare the different methods used. Students may find that they can obtain the answer readily by either method.

4. Students are asked to suggest a method to solve Problem (c). They may suggest using the "trial and error" method or setting up an equation. When comparing the two methods, they may realize that the latter one is better.
   The equation is:
   $$n\,(5x + 8(2x)) = 273$$
   which reduces to
   $$n\,(21x) = 273$$
   i.e.
   $$nx = 13 \quad \text{----------(*)}$$

   The teacher asks students whether the equation can be solved. It seems impossible to solve the equation (*) as there are two unknowns in one equation. If necessary, the teacher may ask them to suggest some possible values of n and x satisfying the equation (*). The solutions are $n = 13$, $x = 1$ or $n = 1$, $x = 13$. The teacher then discusses with students whether the solution is reasonable with reference to the real situation.

5. Problem (d) is more challenging since there are three unknowns in only one equation, namely $n(5x + 8y) = 378$. Students need to make some guesses on the values of the unknowns satisfying the equation. The focus is on how guesses can be proceeded in the right direction. The teacher can suggest students setting up Table 1.1 to investigate the solution.

   Let n be the total number of people in the party, x be the number of cans of soft drinks and y be the number of chicken wings for each person.

   Ask students to guess some values for n, x and y. If necessary, the teacher may give them some hints. Some values of the cost per person such as \$18 or \$21 can be given to them. Then students are asked to find the corresponding value of n and try to list out all the possible values of x and y for each value of n. After that, students are encouraged to extend the table to as many cases as possible. Again, the teacher discusses with students whether the solution is reasonable with reference to the real situation.

Cost per person ($)	***n***	***x***	***y***
\$18	21	2	1
\$21	18		
\$63			
\$378			
\$1			

Table 1.1

6. Possible answers:

Problem	**Possible answers**			
(a)	\$168			
(b)	4 chicken wings			
(c)	$n = 13$ and $x = 2$			
(d)	Cost per person (\$)	n	x	y
	1	378	No integral solution	No integral solution
	2	189	No integral solution	No integral solution
	3	126	No integral solution	No integral solution
	6	63	No integral solution	No integral solution
	7	54	No integral solution	No integral solution
	9	42	No integral solution	No integral solution
	14	27	No integral solution	No integral solution
	18	21	2	1
	21	18	1	2
	27	14	No integral solution	No integral solution
	42	9	2	4
	54	7	6	3
	63	6	3	6
			11	1
	126	3	6	12
			14	7
			22	2
	189	2	1	23
			9	18
			17	13
			25	8
			33	3
	378	1	2	46
			10	41
			18	36
			26	31
			34	26
			42	21
			50	16
			58	11
			66	6
			74	1