**High Order**
  Thinking Skills

*Exemplar 2:*
**Perimeter and Area**

| | |
|---|---|
| **Objectives :** | Students will be able to |
| | (1)  acquire the knowledge of modeling; |
| | (2)  compare the reality with their mathematical model and modify the model; |
| | (3)  investigate the maximum area of a rectangle with a fixed perimeter. |
| | |
| **Dimension :** | Measures, Shape and Space |
| | |
| **Learning Unit :** | More about Areas and Volumes |
| | |
| **Key Stage :** | 3 |
| | |
| **Materials Required :** | Rectangular cards |
| | |
| **Prerequisite Knowledge :** | Perimeter and area of a rectangle |
| | |
| **Main HOTS Involved :** | Problem Solving Skills, Inquiring Skills, Reasoning Skills |

**Problem :**   A class of 40 students need to use their own desks with rectangular tops to enclose a rectangular region. How can they arrange their desks to give a maximum area to the enclosed region?

**Description of the Activity :**

1.   Divide students into groups.

2.   The teacher describes the problem to the class.

3.   Discuss with students the meaning of mathematical modeling and how to construct a model in this case.

4.   Distribute a large rectangular sheet to each group. Ask students to cut out 40 identical rectangular cards from the sheet that represent the desks and allow students to explore the problem with the cards. The cards form the boundary of the enclosed region.

5. Describe the dimensions of the region constructed by completing the table in Worksheet 2.1. The length of the rectangular top of each desk represents a length of unity.

6. The teacher then conducts a discussion:
   (a) What is the perimeter of the rectangle enclosed?
   (b) For a rectangle with fixed perimeter,
       (i) how does its length change as its breadth varies?
       (ii) how does its area change as its breadth varies?
   (c) What is the maximum area of the rectangle and its corresponding dimensions? In such case, what does the shape of this enclosed region become?
   (d) Predict the dimensions of the rectangle which gives the maximum area if the class has 38 students only.

7. Guide students to draw conclusion throughout the discussion.

## Worksheet 2.1

Complete the following table:

| Enclosed rectangular region | | | |
|---|---|---|---|
| **Breadth (in units)** | **Length (in units)** | **Perimeter (in units)** | **Area (in square units)** |
| 2 | 18 | 40 | 36 |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

**Notes for Teachers**

1. Students are guided to observe that the arrangement of the rectangles in Figure 2.1 will not give a maximum area. In fact, the correct arrangement of the rectangles is shown in Figure 2.2.

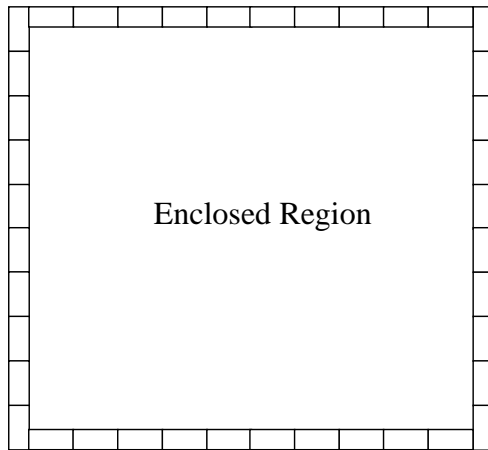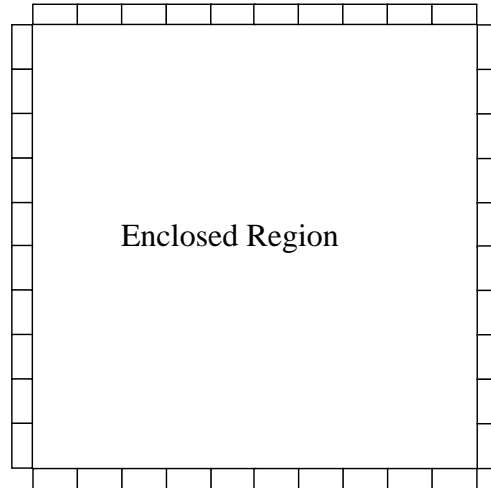

Figure 2.1                                   Figure 2.2

2. From this activity, students will acquire the problem solving skills through the process of constructing a mathematical model (refer to Figure 2.3).
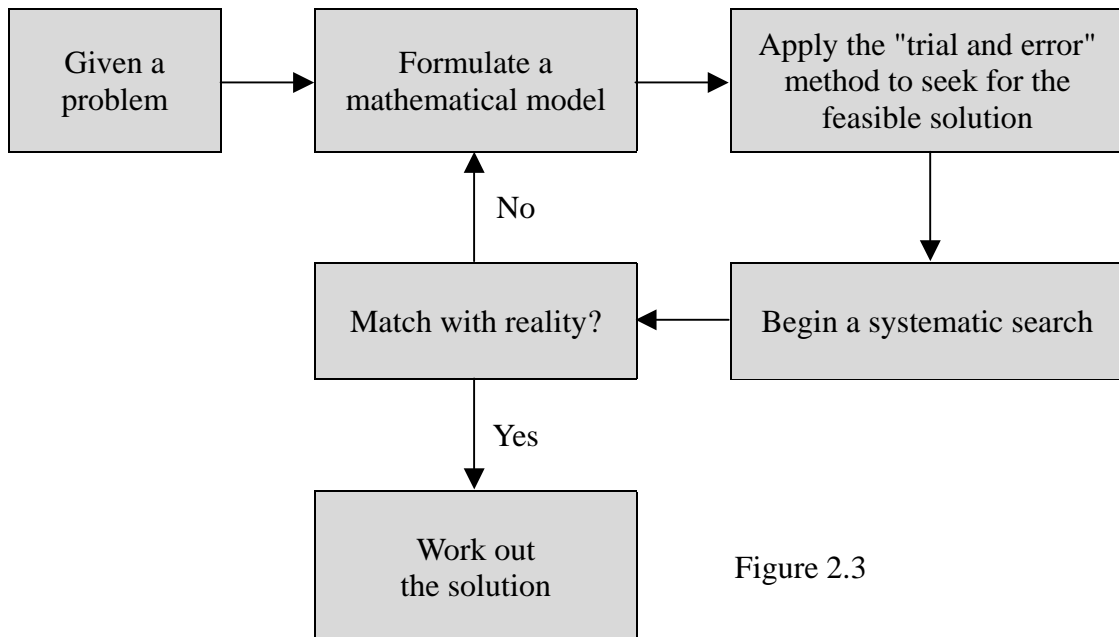


Figure 2.3

3. The perimeter of the rectangular region is constant. The length increases (decreases) as the breadth decreases (increases).

4. The area increases to a certain value and then starts to decrease.

5. For a class of 40 students, the required region is a square with 10 desks along each side. In other words, a rectangle with a fixed perimeter of 40 units has a maximum area when it is a square of side 10 units in length.

6. For a class of 38 students, the required region is a rectangle with 9 desks and 10 desks along the adjacent sides. In other words, if the perimeter of a rectangle with dimensions in whole numbers is 38 units, then its area is maximum when its dimensions are 10 units and 9 units.

7. The following problem is for enrichment:

   Find the least number of desks required for enclosing a rectangular region of area 45 square units if the length of the rectangular top of each desk represents one unit.

   Alternatively, find the least perimeter of a rectangle with dimensions in whole number whose area is 45 square units.