Exemplar 4 :
Pattern

Objectives : Students will be able to
(1) investigate, appreciate and observe the pattern of numbers in Fibonacci sequence;
(2) use algebraic symbols to represent the pattern.

Dimension : Number and Algebra

Learning Unit : Formulating Problems with Algebraic Language

Key Stage : 3

Materials Required : Overhead projector, transparencies, markers and worksheets

Prerequisite Knowledge :
(1) The four basic arithmetic operations of integers
(2) Functional notation

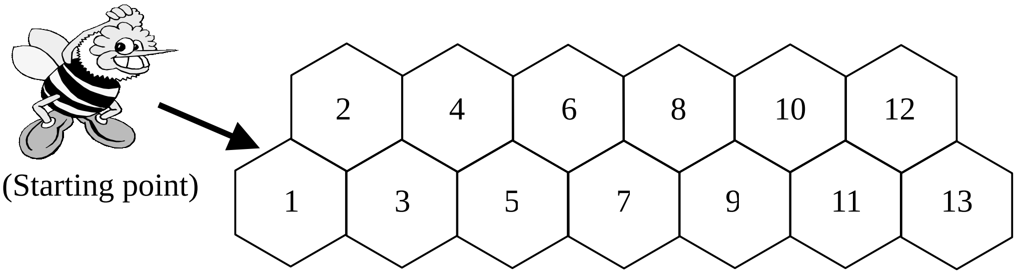
Main HOTS Involved : Inquiring Skills, Reasoning Skills

Description of the activity :

Activity*：*Counting the number of path for a bee

1. Distribute Worksheet 4.1 to students.

2. With the aid of transparency, the teacher describes the problem to students:



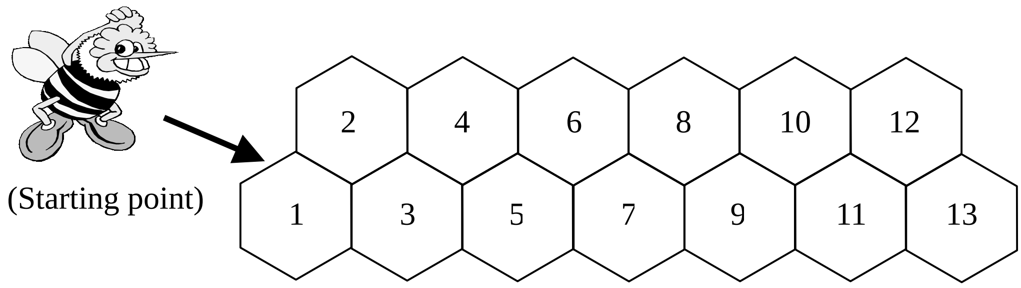
In the figure, a bee starts at cell 1 in its hive and moves to the right only to a cell with a bigger number. Count the number of paths that brings the bee from the starting point to the cell numbering *n* where *n* = 1, 2, 3, ….

3. Ask students to complete Worksheet 4.1.

4. Invite students to give an oral answer.

5. The teacher comments on the answers.

Worksheet 4.1 :



In the figure, a bee starts at cell 1 in its hive and moves to the right only to a cell with a bigger number. Count the number of paths that brings the bee from the starting point to the cell numbering *n* where *n* = 1, 2, 3,……

n	**T(*n*)** **(No. of paths from the starting point to the cell numbering *n*)**
1	
2	
3	
4	
5	
6	
7	
8	

1. What is the relationship among T(1), T(2) and T(3)? Explain the relationship by referring to the movement of the bee.

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2. What is the relationship among T(2), T(3) and T(4)? Explain the relationship by referring to the movement of the bee.

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3. What is the relationship among any three consecutive terms of T(n)? Use T($n-2$), T($n-1$) and T(n) to denote three consecutive terms of the sequence. Explain the relationship by referring to the movement of the bee.

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4. Can you describe the pattern of the number of paths by the relationship in Question 3 only? If not, what other conditions must be added to describe the whole pattern?

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5. Write down all the conditions that are sufficient to describe the pattern of the number of paths from the starting point to the cell numbering n.

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Notes for Teachers :

1. The teacher should make sure that students can use the functional notation, T(n), to denote the nth term in a sequence.

2. The number of paths from the starting point to the cell numbering n, T(n), is given by the following conditions:
   (a) T(1) = 1;
   (b) T(2) = 2; and
   (c) T(n) = T(n – 2) + T(n – 1) for n > 2 and n is a natural number.

3. Students may not find it easy to explain why condition (c) holds in general. Guidance from the teacher should be offered.
   (a) The teacher may guide students to consider the following questions:
      (i) Which cell does the bee reach before entering the cell numbering n?
      Answer: Cell numbering n – 2 or cell numbering n – 1.
      (ii) How many paths are there from the cell numbering n – 1 to the cell numbering n and from the cell numbering n – 2 to the cell numbering n?
      Answer: There is only one path for either case.
      (iii) How many paths are there from the starting point to the cell numbering n via the cell numbering n – 2 and via the cell numbering n – 1?
      Answer: T(n – 2) for the first part and T(n – 1) for the second part.
   (b) The teacher then helps students conclude that the number of paths for the bee going from the starting point to the cell numbering n is the sum of the number of paths from the starting point to the cell numbering n – 2 and the number of paths from the starting point to the cell numbering n – 1.

4. Students should be told that any sequence satisfying the conditions give in (2) are in Fibonacci sequence.

5. T(n) may also be defined by

$$T(n) = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right]$$

for $n \geq 1$.