Exemplar 5：
To Investigate the Properties of Trapeziums

Objectives : Students will be able to explore the properties of trapeziums through a computer software.

Dimension : Measures, Shapes and Space

Learning Unit : Quadrilaterals

Key Stage : 3

Material Required : Dynamic geometry software - *Geometer's Sketchpad* and the files hots05_1.gsp, hots05_2.gsp and hots05_3.gsp

Prerequisite Knowledge : Area of a triangle

Main HOTS Involved : Inquiring Skills, Reasoning Skills

Description of the Activity :

1. Distribute Worksheet 5.1 and the files hots05_1.gsp, hots05_2.gsp and hots05_3.gsp to students.

2. Ask students to investigate the problems in the worksheet. Write down their findings.

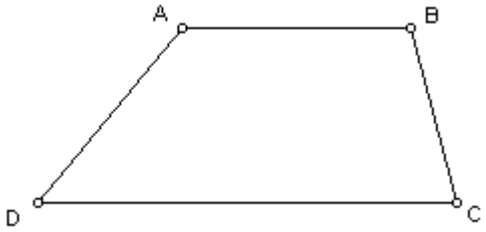
3. Ask students to compare and discuss their results with their classmates. To present their findings, they could print out the related sketches showing different trapeziums and the measures of the sides or angles.

4. The teacher gives comments on their findings.

Worksheet 5.1 :

Activity 5.1 :

Open the file hots05_1.gsp to get the trapezium ABCD, where AB is the upper base and CD is the lower base. Drag the vertices to form trapeziums of different sizes and shapes. By measuring the lengths of the sides and the sizes of the angles, investigate the following：



1. Can AB or CD be equal to AD or BC? If so, draw a figure on the screen, otherwise, explain it.

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2. Can AC = BD? If so, draw a figure on the screen. What can you observe about the lengths of AD and BC?

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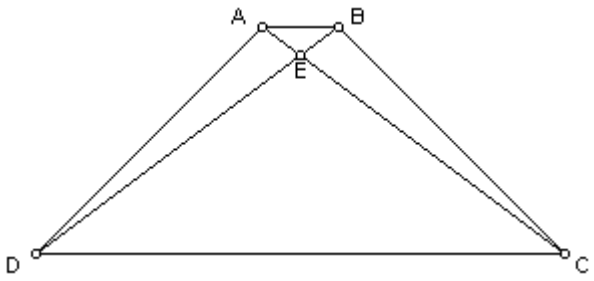
Activity 5.2 :

Open the file hots05_2.gsp to get an isosceles trapezium ABCD. E is the point of intersection of the diagonals AC and BD.



3. Measure ∠BCD and ∠ADC; ∠BAD and ∠ABC. What do you find?

   _____

4. Measure ∠BAD and ∠ADC; ∠ABC and ∠BCD. What do you find?

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5. Find the areas of ΔAED and ΔBEC. What do you observe?

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6. Find the areas of ΔAED, ΔAEB, ΔBEC and ΔDEC. What is the relationship among the areas?

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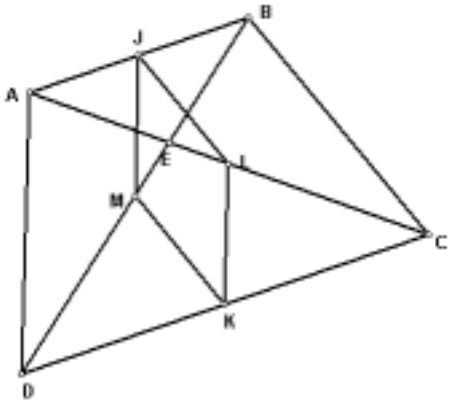
Activity 5.3 :

7. Open the file hots05_3.gsp. J, K, L and M are the midpoints of AB, CD, AC and BD respectively.



What can you say about the figures formed by joining these four midpoints or any three of them? State your conjectures. Verify it by measuring the lengths of the quadrilateral JKLM. How are these figures changed if the trapezium is no longer an isosceles trapezium?

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Notes for Teachers :

1. The demo version of *Geometer's Sketchpad* can be downloaded from the web site http://www.keypress.com/sketchpad/sketchdemo.html

2. The teacher should load the files hots05_1.gsp, hots05_2.gsp and hots05_3.gsp in the computer server for students to use. Alternatively, the teacher may load the files on floppy diskettes.

3. The teacher should explain to students the meaning of base and isosceles trapezium. The pair of parallel sides are called the bases. If the non-parallel sides have equal length, then it is called isosceles trapezium.

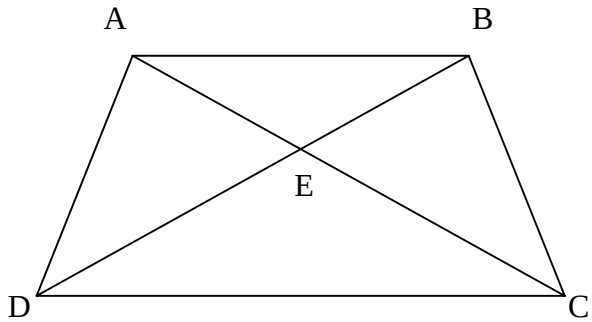
4. In the second question of Worksheet 5.1, students are expected to discover that AC = BD if AD = BC.

5. In Question 3, by measuring the sizes of the base angles (e.g. $\angle$BCD and $\angle$ADC), students should find that the angles are always equal no matter how they change the size of the isosceles trapezium.

6. In Question 4, they are expected to discover that the two pairs of angles are supplementary.

7. In Question 5, students should be able to observe that ΔAED and ΔBEC are equal in area.

8. In Question 6, one way to describe the relation is
   area of ΔAED $\times$ area of ΔBEC = area of ΔAEB $\times$ area of ΔDEC.



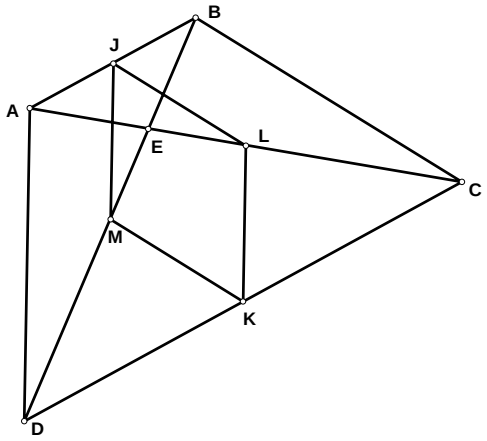
Proof : $\dfrac{\text{area of } \Delta AED}{\text{area of } \Delta AEB} = \dfrac{DE}{EB} = \dfrac{\text{area of } \Delta DEC}{\text{area of } \Delta BEC}$

$\therefore$ area of ΔAED $\times$ area of ΔBEC = area of ΔAEB $\times$ area of ΔDEC.

The teacher should alert students that this relation is valid for any quadrilateral.

9. Question 7 is a more challenging problem. The teacher can help students form small groups and encourage them to discuss their conjectures. If the trapezium is isosceles, then the quadrilateral formed by joining the four midpoints is a rhombus.

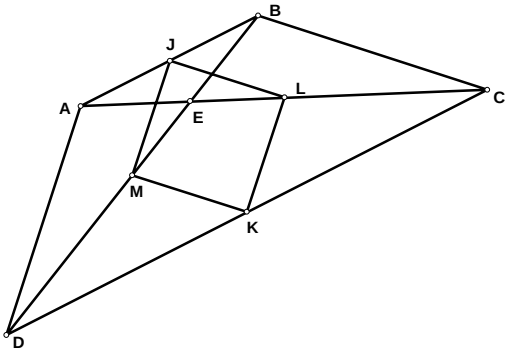
JL = 2.53 cm
LK = 2.53 cm
KM = 2.53 cm
MJ = 2.53 cm
∠JMK = 121°



The teacher can ask students to have further exploration. For example, when will the quadrilateral JLKM become a square? In which situation the figure cannot be formed?

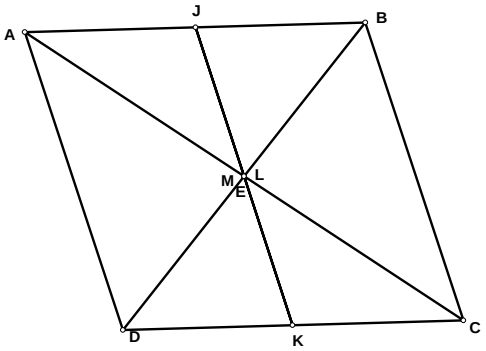
When the base angles on one side of the trapezium are 45°, the quadrilateral JLKM will become a square.

JL = 2.17 cm
LK = 2.17 cm
KM = 2.17 cm
MJ = 2.17 cm
∠JMK = 90.0°
∠MKL = 90.0°
∠ADC = 45.0°



If the trapezium becomes a parallelogram, rectangle or square, the figure JLKM cannot be formed.

JL = 2.94 cm
LK = 2.94 cm
KM = 2.94 cm
MJ = 2.94 cm
∠JMK = 180.0°
∠MKL = 0.0°
∠ADC = 106.6°



For any set of three midpoints, the figure formed by joining them can be an isosceles triangle or an equilateral triangle.

If the trapezium is not isosceles, the four midpoints J, K, L, and M can form a parallelogram or a rectangle. For any set of three midpoints, the figure formed can be an acute-angled, obtuse-angled or right-angled triangle.