## High Order

Thinking Skills


Objectives :

## Exemplar 6 : A Game of Dice

Students will be able to

1. investigate probabilities in real-life activities;
2. consolidate the relation between empirical and theoretical probabilities.

Dimension :
Data Handling

Learning Unit : Simple Idea of Probability

Key Stage :
3

Materials Required : Dice, transparencies and overhead projector

Prerequisite Knowledge : Meaning of probability, calculations of empirical and theoretical probabilities

Main HOTS Involved : Inquiring Skills

## Description of the Activity :

1. Divide students in pairs. One of them is called "Mr. L" and the other is "Mr. H".
2. Assign a group number to each pair and give them two dice.
3. Ask students to perform the following game :
(a) Throw two dice 25 times.
(b) In each throw, subtract the smaller number from the larger. The answer is called "Dice Difference".
(c) If the "Dice Difference" is 0,1 or 2, then Mr. L wins. If the "Dice Difference" is 3,4 or 5 , Mr. H wins.
4. Instruct each group to enter their results in Worksheet 6.1 provided.
5. Ask students to complete Table 6.1 printed on a transparency:

| Group number | Number of times Mr. L wins | Number of times Mr. H wins |
| :---: | :---: | :---: |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |
| 13 |  |  |
| 14 |  |  |
| 15 |  |  |
| 16 |  |  |
| 17 |  |  |
| 18 |  |  |
| 19 |  |  |
| 20 |  |  |
| Total |  |  |

Table 6.1
6. Discuss the following questions with students:
(a) What is the empirical probability that Mr. L wins in each group? (The teacher can add one more column to Table 6.1 and write down the answers.)
(b) Is there any pattern found in (a)?
(c) Does the game seem to be fair to both players? Why?

At this moment, the teacher does not make any comments on students' findings.

## High Order

Thinking Skills
7. Ask students to use the data obtained in Table 6.1 to complete Worksheet 6.2.
8. Discuss the following questions with students:
(a) What will the empirical probability that Mr . L wins tend to in the long run?
(b) What will the empirical probability that $\mathrm{Mr} . \mathrm{H}$ wins tend to in the long run?
(c) Is the game fair to both players? What conclusion can be drawn?
9. By using the grid paper or any other method, list out the sample space of the outcomes of throwing two dice and then determine the numbers of favorable outcomes (i.e. Mr. L or Mr. H wins).
10. Students are guided to draw conclusions from the theoretical probability that each player wins and determine whether the game is fair or not.
11. If the game is not fair, ask students to modify the rules and make it fair.
12. Give justifications for their suggestions.

## Worksheet 6.1

Please tick the appropriate box.

| No. of Throw | Mr. L wins | Mr. H wins |
| :---: | :---: | :---: |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |
| 13 |  |  |
| 14 |  |  |
| 15 |  |  |
| 16 |  |  |
| 17 |  |  |
| 18 |  |  |
| 19 |  |  |
| 20 |  |  |
| 21 |  |  |
| 22 |  |  |
| 23 |  |  |
| 24 |  |  |
| 25 |  |  |
| Total no. of wins |  |  |

Who is the winner?
6.4

## Worksheet 6.2

Please complete the following table.

| Group number less than or equal to | Accumulated number of times Mr. L wins | Number of rounds played | Empirical probability that Mr. L wins |
| :---: | :---: | :---: | :---: |
| 1 |  | 25 |  |
| 2 |  | 50 |  |
| 3 |  | 75 |  |
| 4 |  | 100 |  |
| 5 |  | 125 |  |
| 6 |  | 150 |  |
| 7 |  | 175 |  |
| 8 |  | 200 |  |
| 9 |  | 225 |  |
| 10 |  | 250 |  |
| 11 |  | 275 |  |
| 12 |  | 300 |  |
| 13 |  | 325 |  |
| 14 |  | 350 |  |
| 15 |  | 375 |  |
| 16 |  | 400 |  |
| 17 |  | 425 |  |
| 18 |  | 450 |  |
| 19 |  | 475 |  |
| 20 |  | 500 |  |

## Notes for Teachers :

1. Throughout the game, students will experience that the empirical probability will get close to the theoretical probability as the number of trials increases.
2. Students can use (a) a grid paper; (b) ordered pairs; (c) the tabulation method to show all possible outcomes when two dice are thrown.
(a) Grid paper

$1^{\text {st }}$ die
(b) Ordered pairs

|  |  |  |  |  |  |  |  | $1^{\text {st }}$ die |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |  |  |  |  |  |  |  |  |
| $2^{\text {nd }} d i e$ | $(1,1)$ | $(2,1)$ | $(3,1)$ | $(4,1)$ | $(5,1)$ | $(6,1)$ |  |  |  |  |  |  |  |  |
| $\mathbf{1}$ | $(1,2)$ | $(2,2)$ | $(3,2)$ | $(4,2)$ | $(5,2)$ | $(6,2)$ |  |  |  |  |  |  |  |  |
| $\mathbf{2}$ | $(1,3)$ | $(2,3)$ | $(3,3)$ | $(4,3)$ | $(5,3)$ | $(6,3)$ |  |  |  |  |  |  |  |  |
| $\mathbf{3}$ | $(1,4)$ | $(2,4)$ | $(3,4)$ | $(4,4)$ | $(5,4)$ | $(6,4)$ |  |  |  |  |  |  |  |  |
|  | $\mathbf{5}$ | $(1,5)$ | $(2,5)$ | $(3,5)$ | $(4,5)$ | $(5,5)$ |  |  |  |  |  |  |  |  |
| $(6,5)$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\mathbf{6}$ | $(1,6)$ | $(2,6)$ | $(3,6)$ | $(4,6)$ | $(5,6)$ | $(6,6)$ |  |  |  |  |  |  |  |  |

## High Order

Thinking Skills
Exemplar 6
(c) Tabulation method

| Dice Difference | Favorable outcomes in ordered pairs |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- |
| 0 | $(1,1)$ | $(2,2)$ | $(3,3)$ | $(4,4)$ | $(5,5)$ |$(6,6)$

3. Study the following table.

| Dice Difference | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total number of <br> favorable outcomes | 6 | 10 | 8 | 6 | 4 | 2 |

This game is not fair since the theoretical probability of Mr. L wins is $\frac{6+10+8}{36}=\frac{2}{3}$ and that of Mr. H is $\frac{6+4+2}{36}=\frac{1}{3}$.
4. Suggestion for modifying the rules to make the game more fair:

Replace Mr. L and Mr. H by Mr. E and Mr. O. Mr. E wins if the dice difference is 0 , 2 or 4 and Mr . O wins if the dice difference is 1,3 or 5 .
Justification:
$\mathrm{P}(\mathrm{Mr}$. E wins $)=\frac{6+8+4}{36}=\frac{1}{2} \quad \mathrm{P}($ Mr. O wins $)=\frac{10+6+2}{36}=\frac{1}{2}$
Accept other modifications from students provided that the two players have equal chance of winning.

