Exemplar 7 :
Addition and Subtraction of Simple Polynomials

Objective : Students will be able to add and subtract simple polynomials.

Dimension : Number and Algebra

Learning Unit : Manipulations of Simple Polynomials

Key Stage : 3

Materials Required : Tiles (or cards if tiles are not available)

Prerequisite Knowledge : (1) Four basic operations of directed numbers
(2) Basic concepts of Algebra

Main HOTs Involved : Conceptualizing Skills, Communicating Skills

Description of the Activities :

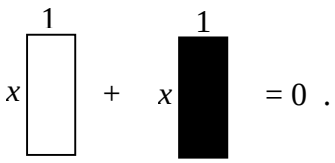
Activity 7.1 : Addition of simple polynomials.

1. Before the lesson, the teacher prepares enough tiles with the following dimensions.

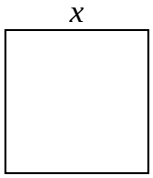
		Meaning		Meaning		Meaning
White tile	x (top), x (left)	$+x^2$	1 (top), x (left)	$+x$	1 (top), 1 (left)	$+1$
	x (top), x (left)	$-x^2$	1 (top), x (left)	$-x$	1 (top), 1 (left)	-1

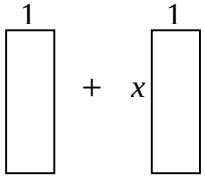
2. The Teacher displays the tiles to the whole class and clarifies the meaning of each tile (i.e. each term of the polynomial is represented by a particular tile).

Students should note that each white tile cancels a black tile of the same dimension.

For example, 

3. The teacher divides students into groups of 2 and asks them to state the differences in tile representation between x^2 and $2x$.

(Answer: x^2 : 

$2x$: 

)

Problems : (i) What is the difference between these two representations?
(ii) What will these representations be if x is now changed to $2x$?

4. The teacher distributes enough number of tiles to each group and asks students to use them to represent polynomials $3x + 2$ and $x^2 + 4x + 1$.

5. One student from each group is responsible for writing down different polynomials of the second degree on the paper and the other student of the same group uses suitable tiles to represent the polynomials. Students swap their roles after a few successful trials.

6. The teacher can now demonstrate the way to add two trinomials by putting their corresponding tile representations together and sort them in accordance to their shapes and sizes, and consequently yield the answer.

7. In order to facilitate the learning of adding different polynomials by tiles, the teacher can put two groups together so that one group will design questions on the addition of trinomials and the other group will use tile representations to do calculations. Students swap their roles after sufficient practices.

8. After all students have made enough practices, the teacher may ask them to complete Worksheet 7.1.

9. Discussion within each group is encouraged. Write down the answers on the worksheet.

10. The teacher may go through the answers of these questions with students.

11. Further problem: This representation works well for polynomials up to degree 2. How do we represent polynomials of degree 3 in a similar way?

Worksheet 7.1

1. For question (a) in Table 7.1 below, use tiles to represent the given algebraic expression in the cell numbering ① and draw the corresponding tiles representation in the cell numbering ②.

2. Simplify the result and draw the tile representation in the cell numbering ③. Write down the corresponding algebraic expression in the cell numbering ④.

3. Complete questions (b) to (d).

4. Discuss with other group members.

Table 7.1

Qn		Original	New
(a)	Algebraic expression	① $(3x + 2) + (2x + 1)$	④
	Tile representation	②	③
(b)	Algebraic expression	$(3x + 2) + (2x - 5)$	
	Tile representation		

(to be continued, PTO)

Table 7.1 (continued)

Qn		Original	New
(c)	Algebraic expression	$(3x + 2) + (-x + 1)$	
	Tile representation		
(d)	Algebraic expression	$(3x + 2) + (-x - 3)$	
	Tile representation		

5. Discuss with other group members on how to simplify the expression

$$(ax + b) + (cx + d).$$

6. Simplify the following expressions. You need to write down the final polynomials only.
   (a) $(2x + 1) + (-3x - 3)$
   (b) $(x^2 + 2x + 3) + (2x^2 - x + 1)$
   (c) $(2x^2 - 1) + (x + 1)$
   (d) $(2 - x + x^2) + (x - x^2)$
   (e) $(2x^2 + x - 3) + (-2x^2 - x + 3)$

Activity 7.2 : Subtraction of simple polynomials

1. Distribute a number of different kinds of tiles to each group.

2. Distribute Worksheet 7.2 to students.

3. Ask each group to discuss question 1 of Worksheet 7.2 and finish it. The teacher discusses question 1(a) with students.

4. (a) Write down the expression $(3x + 2) – (2x + 3)$ on the blackboard.
   (b) Ask students to make use of tiles to simplify it.
   (c) As students may find that there is not enough square tiles of dimension 1×1 for subtraction, the teacher can ask each group to conduct a discussion to solve the problem. Each group reports their solutions to the whole class. The teacher and other students can comment on their solutions.

5. Ask students to make use of tiles to finish questions 2 to 4 in Worksheet 7.2. Students need to discuss with their group members.

6. Students need to report their observations in question 4 of Worksheet 7.2 to the whole class.

7. The teacher and other students make comments.

8. Ask students to work on question 5 individually.

9. The teacher checks the answers with students.

Worksheet 7.2

1. Make use of the tiles to simplify the following expressions.
   (a) $(4x + 3) – (2x + 1)$
   (b) $(2x^2 + x + 3) – (x^2 + x + 2)$
   (c) $(–3x + 2) – (–2x + 2)$

2. Make use of the tiles to simplify the following expressions.
   (a) $(3x + 1) – (2x + 2)$
   (b) $(3x + 1) – (2x – 2)$
   (c) $(3x + 1) – (–2x + 2)$
   (d) $(3x + 1) – (–2x – 2)$

3. Make use of the results you obtained in question 2 to do the matching below.

Expression in Question 2			Expression
$(3x + 1) – (2x + 2)$	•	•	$3x + 1 + 2x – 2$
$(3x + 1) – (2x – 2)$	•	•	$3x + 1 –2x – 2$
$(3x + 1) – (–2x + 2)$	•	•	$3x + 1 + 2x + 2$
$(3x + 1) – (–2x – 2)$	•	•	$3x + 1 –2x + 2$

4. From the above results of matching, what do you observe?
   Write down any rule(s) that you observe below.
   e.g. $(ax + b) – (cx + d) = ?$

   _____

   _____

   _____

5. Simplify the following expressions.
   (a) $(2x + 1) – (–3x –3)$
   (b) $(2x^2 + 2x + 3) – (x^2 – x + 1)$
   (c) $(2x^2 – 1) – (x + 1)$
   (d) $(2 – x + x^2) + (x – x^2)$
   (e) $(2x^2 + x –3) – (–2x^2 – x + 3)$
   (f) $(2x^2 – 3 – 2x) + (4 – 4x^2 + 5x)$

Notes for Teachers :

Activity 7.1 :

1. The first part of the activity aims to clarify the meaning of the tile representation and helps students represent a simple polynomial by tiles.

2. For the second question in point 3 of Activity 7.1, if x is replaced by $2x$, the square will be four times as large as the original square; and the representation of $2(2x)$ is twice as large as that of $2x$.

3. For further consolidations of representing polynomials by tiles, the teacher can stick a list of different sets of tiles on the blackboard and ask students to write down their corresponding polynomials. This helps students familiarize the one-one correspondence between a set of tiles and its corresponding polynomial.

4. When students attempt Worksheet 7.1, they have to make use of communicating skills such as using mathematical language to express their ideas, writing down their observations through discussion. This worksheet helps students group like terms in order to simplify the expressions. The verbal presentation of each group fosters communication among students. As a result, the observation in question 5 of Worksheet 7.1 is suggested as

   "$(ax + b) + (cx + d) = (ax + cx) + (b + d) = (a + c)x + (b + d)$".

5. In question 6 of Worksheet 7.1, students are expected to apply the technique of grouping like terms (learned in question 5) to simplify the expressions without making use of the tiles.

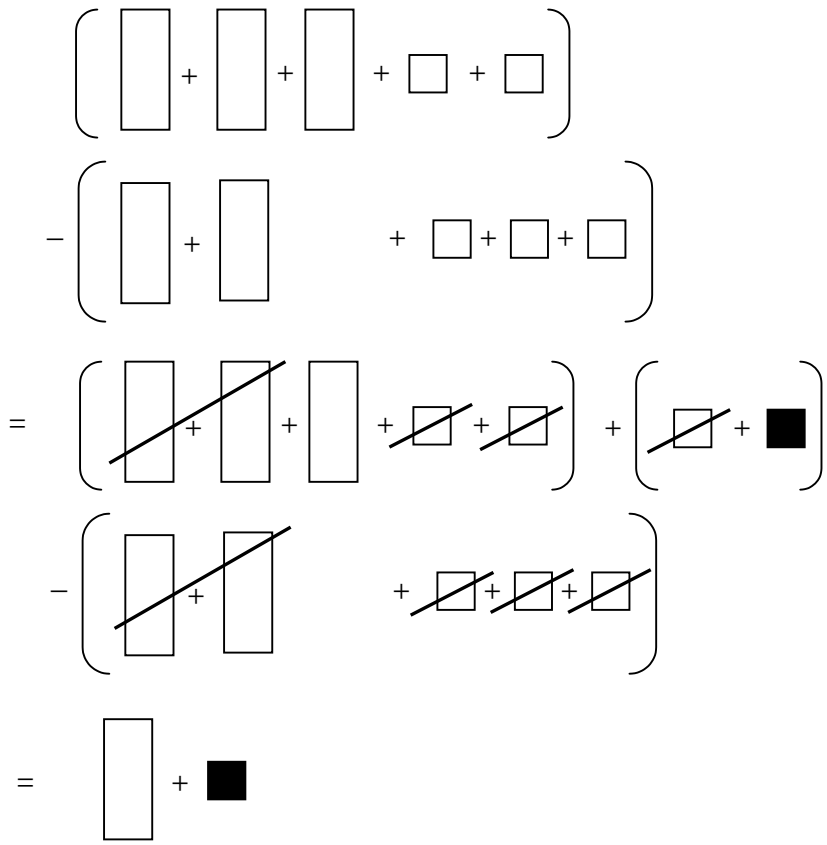
6. For point 11 in the description of Activity 7.1, one feasible answer is to represent x^3 by a cube of dimensions x units $\times$ x units $\times$ x units, x^2 by a block of dimensions x units $\times$ x units $\times$ 1 unit and so on.

Activity 7.2 :

1. Students should be able to discover that subtraction means taking away tiles from the tile representation of the first polynomial. The teacher should note that there are enough tiles for removal in question 1.

2. The method in question 1 cannot be used to solve question 2 because there are insufficient tiles to be taken away. Students should consider a new method in tackling this problem. The teacher can either lead the discussion or let students discuss by themselves. The answer is just adding enough zeros (a white tile and a black tile to make a zero) until you get enough tiles for removal when doing subtraction.

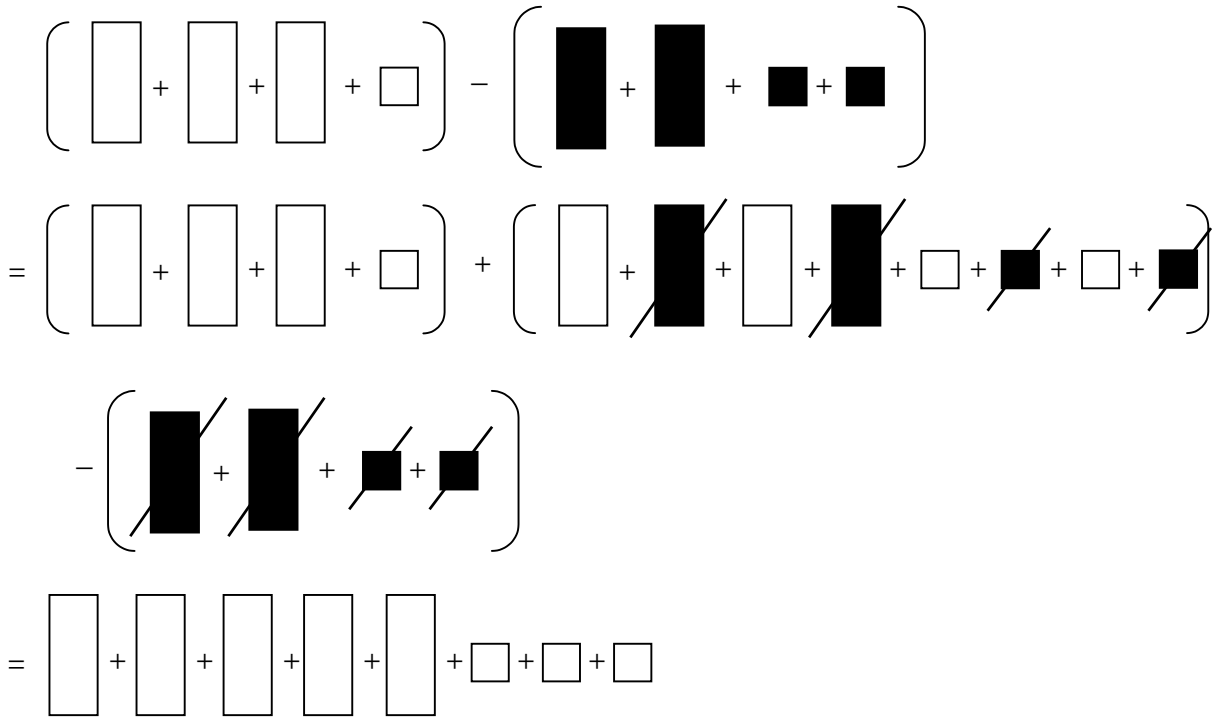
   e.g. $(3x + 2) – (2x + 3)$ can be represented by



   The answer is $x – 1$.

e.g. $(3x + 1) – (– 2x –2) = (3x + 1) – [2(–x) + (–2)]$

which can be represented by



The answer is $5x+3$.

3. The use of questions 2 and 3 of Worksheet 7.2 helps students discover the distributive rule for subtraction. i.e. $(ax + b) – (cx + d) = ax + b – cx – d$.

4. Students are expected to use the rule they have discovered in question 4 of Worksheet 7.2 to solve question 5 without making use of the tiles.