Five High Order Thinking Skills

Introduction

The high technology like computers and calculators has profoundly changed the world of mathematics education. It is not only what aspects of mathematics are essential for learning, but also how mathematics is done and what attitude towards mathematics learning is fostered. Therefore, apart from mathematical content, thinking processes and attitude are also essential core components for mathematics learning at various stages of schooling.

Teaching high order thinking skills (HOTS) is currently at the centre of educational attention. In particular, the revised secondary mathematics curriculum has shifted its emphasis to the fostering of HOTS. In general, measures of high order thinking include all intellectual tasks that call for more than the retrieval of information. Therefore, in broad terms, HOTS can be considered as the skills required for performing these tasks. Five fundamental HOTS have been identified in the Syllabus. They are: problem solving skills, inquiring skills, reasoning skills, communicating skills and conceptualizing skills. These fundamental and intertwining ways of learning mathematics, thinking and using mathematical knowledge are considered important in mathematics education. In fact, many of students' problems in learning mathematics originate from their weaknesses in one or more of these skills. Students are expected to enhance the development of these skills and use them to construct their mathematical knowledge, and hence engage in life-long learning.

Teachers should note the following four points. First, there is no simple, clear and universally accepted definition of HOTS. In fact, they may be arranged into several overlapping categories such as metacognitive skills, critical and creative thinking. Nevertheless, it is generally agreed that high order thinking is non-algorithmic and complex; it involves self-regulation of the thinking process and often yields multiple solutions to tasks. This generally agreed features of HOTS give rise to the common technique of posing open-ended problems for fostering HOTS in mathematics classes.

This technique gives students chances to think about mathematics and to talk about mathematics with each other and with their teacher.

Second, the five HOTS cannot be easily isolated from each other in mathematical work. For example, a student using reasoning skills to solve a problem may also be considered as demonstrating his/her problem solving skills. Similarly, communicating skills are always involved in doing mathematical tasks, and conceptualizing skills are engaged in all exploratory work. Therefore, the exemplars compiled in this teaching package only demonstrate the main HOTS which can be developed through the accomplishment of the exemplars concerned.

Third, HOTS can be taught in isolation from specific contents, but incorporating them into content areas seems to be a popular way of teaching these skills. The key to generalizing HOTS into a content area is for the student to focus on a skill that was learned in one setting and to see its relevance in another.

Fourth, computer provides an excellent tool for teaching HOTS because of its interactive capabilities and its ability to present and stimulate problems. The computer is an effective medium to help teach the skills initially and to promote generalizations. Using a computer enables students to spend more time on the actual thinking tasks rather than on peripheral and trivial activities. It indirectly enhances thinking skills by making it possible for students to spend their overall time more effectively.

Problem Solving Skills

Problem solving is an integral part of all mathematics learning and it involves identifying obstacles, constraints or unexpected patterns, trying different procedures and evaluating or justifying the solution. The National Council of Teachers of Mathematics (NCTM) [1] considers problem solving as a process of applying previously acquired knowledge to new and unfamiliar (or unforeseen) situations. To

[1] NCTM (1989). *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va.: National Council of Teachers of Mathematics.

solve a problem, students draw on their knowledge and develop new mathematical understandings. They should also acquire ways of thinking, develop confidence and habits of persistence in unfamiliar situations through the problem solving process.

The general problem solving strategies embrace understanding the problem, devising a plan of solving the problem, carrying out the plan, examining the reasonableness of the result and making evaluation[2]. These four phases have formed a framework for problem solving in many mathematics textbooks.

Successful problem solving involves the process of coordinating previous mathematical knowledge and experience to develop a solution of a problem for which a procedure for determining the solution is not known. Intuition may also be involved in the thinking process. Therefore, in the problem solving process, students may make conjectures and try many different ways to tackle the problem. Teachers should note that any method, which can be properly used to solve a problem is a "correct" method. Teachers should not discourage a student merely because his/her method is too long or too complicated. Instead, teachers should guide the student with patience to examine the process or method he/she adopts.

Both non-routine and open-ended problems give students more opportunities to demonstrate their problem solving skill. However, teachers should note that problems of the similar type, even they are non-routine or open-ended problems, when done repeatedly, will become routine problems and hence lose their function of fostering students' HOTS.

Teachers should note that problem solving is more than solving problems. The latter involves strategy of writing down the rule (or formula), demonstrating how to use the rule and providing exercises to students for practising the rule. The former, on the other hand, emphasizes on heuristic processes, developing flexibility and creativity in applying mathematical ideas and skills to unfamiliar questions. Students can acquire opportunities to develop their interest in mathematics and foster their capability of independent thinking through problem solving.

[2] Polya, G. (1957). *How to Solve It*. New York: Doubleday & Co., Inc.

Inquiring Skills

Inquiring involves discovering or constructing knowledge through questioning or testing a hypothesis. Observation, analysis, summarizing and verification are the essential elements in carrying out inquiring activities. Inquiring activities mainly involve self-learning processes, but suitable guidance from teachers are sometimes necessary depending on the abilities of students and the complexity of the activities. Posing questions is one popularly adopted means to guide students to make exploration. In fact, well-designed questions are useful to stimulate students to discover similarities, differences, patterns and trends. Students may also be asked to test mathematical conjectures, which enable them to participate in a more active role in the learning process.

Inquiring activities designed should cope with the abilities of students so that they can enjoy the discovery of mathematical results. Moreover, it may be more effective to arrange students in small groups (whenever possible) because it is easier for them to put forward their ideas. The following list of verbs may be helpful in guiding students to perform inquiring activities: explore, discover, create, prove, validate, construct, predict, experiment, investigate, etc.

Inquiring activities usually requires some teaching aids. Teachers should therefore make proper preparations well before the lesson so that adequate sets of aids are available. The following questions should be considered before organizing the inquiring activities in class:

- Will students be grouped when performing the activities? If yes, how many groups should be organized?
- How can we ensure that the right amount of guidance (in the form of hints or questions) is provided? (It should be noted that either insufficient or too much guidance will do no good to students.)
- When computers are available, what software could be used? Is there sufficient software for the whole class? If no, what can be done?

Communicating Skills

Communication involves receiving and sharing ideas and can be expressed in the forms of numbers, symbols, diagrams, graphs, charts, models and simulations. It is viewed as an integral part of mathematics instruction as it helps clarify concepts and build meaning for ideas. Through the communication process, students learn to be clear and convincing in presenting their mathematical ideas, which definitely help develop their logical thinking.

Since mathematics is very often conveyed in symbols, oral and written communication about mathematical ideas are often overlooked by teachers. However, it should be noted that both oral and written language are needed to describe, explain and justify mathematical ideas. These abilities can help students clarify their thinking and sharpen their understanding of concepts and procedures. Furthermore, during the process of communicating, students may construct, refine and consolidate their mathematical understandings.

Among all forms, written communication is of special importance because it provides students with a record of their own thinking and ideas. Moreover, the process of writing in mathematics learning promotes students' active involvement. Nevertheless, students should be reminded to write concisely, precisely and neatly as mathematics needs clear, consistent, concise and cogent language.

Communication can be fostered in many ways. For example, students can be asked to describe a practical task and to tell what characteristics they discover. Investigative activities and project work are ideal tasks for developing students' communicating skill. Open-ended questions that allow students to construct their own responses and encourage divergent or creative thinking furnish fertile areas for communication. Small group discussions and debates are also helpful. They can be used to encourage students to read, write and discuss their mathematical ideas. However, teachers should pay attention to the suitable arrangement of classroom and groupings for facilitating students to share their ideas. Small collaborative groups afford opportunities to explore ideas while whole-class discussions can be used to compare and contrast ideas from individual students.

Reasoning Skills

Reasoning is drawing conclusions from evidence, grounds or assumptions. It involves developing logical arguments to deduce or infer conclusions. Reasoning may be classified into inductive reasoning and deductive reasoning. Inductive reasoning works from specific observations to broader generalizations and theories while deductive reasoning moves from the other way round, that is, from the more general to the more specific. By its very nature, the inductive reasoning method is more open-ended and exploratory and the deductive one is narrower in nature and is usually concerned with testing or verifying hypotheses and theories. Therefore, finding the general term of a sequence like 1, 3, 5, 7, 9, …… involves inductive reasoning while doing a geometric proof by applying a geometrical theorem (say, the corresponding angles of two similar triangles are equal) involves deductive reasoning.

Since reasoning is a fundamental aspect of mathematics, being able to reason is essential to the understanding of mathematical concepts. By making investigations and conjectures, developing and evaluating mathematical arguments, justifying results, etc., students are able to understand and appreciate the power of reasoning and produce proofs, which entail logical deductions of conclusions from theories and hypotheses. Reasoning, like other HOTS, cannot be taught in a single lesson. Instead, it is a habit of mind and should be a consistent part of students' mathematical experience. It is fostered or developed through a prolonged learning of mathematics in different contexts.

To develop reasoning skills, students should be familiar with the following:

1. Sorting and classifying information, interpreting information and presenting results with pictures, diagrams, graphs, models, symbols and tables.
2. Describing, generalising, justifying patterns presented in a variety of forms and contexts, making conjectures, thinking flexibly, proving and refuting, recognising logical and illogical arguments, following a chain of reasoning, making deductions and demonstrating methods of mathematical proof (including proof by contradiction, counter-example and induction).

It should be noted that reasoning involves informal thinking, making hypotheses and validating them. Students should be encouraged to justify their answers and solution processes. Questions which are useful to help students develop their reasoning skills include:

- Why do you think it is true/false?
- If we go in this way, what happens? How do you know?
- If we change angle A to 90 degrees, will the result remain the same?
- If the lines are not parallel, the theorem is not true. Why?
- Pythagoras' theorem is true for any triangle. Comment.

Conceptualizing Skills

Conceptualizing involves organizing and reorganizing of knowledge through perceiving and thinking about particular experiences in order to abstract patterns and ideas and generalize from the particular experiences. The formation of concepts involves classifying and abstracting of previous experiences.

The particular problem of mathematics lies in its abstractness and generality. Abstract concepts cannot be communicated to students by a definition but only by arranging for him/her to encounter a suitable collection of examples. It follows that abstract concepts should be backed up by an abundance of mathematical and daily-life examples. Teachers need to provide students with a clear guidance to construct mathematical concepts from the examples and use these concepts to solve problems in unfamiliar situations.

When a new concept is introduced (like "All symmetrical triangles are similar."), examples or counter-examples may be provided for illustration. Students may also be asked to explore the information relevant to the concept (like similar triangles or symmetrical triangles) and classify the similarities and differences in the examples.

In short, to help students build up mathematical concepts, suitable examples or activities, which allow students to construct new concepts independently are necessary.