Exemplar 

To Study the Effects of Dilation and Contraction on 2-D Figures

Objective： To recognize the effects of dilation and contraction on the properties of 2-D geometric figures

Dimension： Measures, Shape and Space

Learning Unit： Transformation and Symmetry

Key Stage： 3

Material Required： *Cabri Geometry II*

Prerequisite Knowledge： Basic concepts of triangles

Description of the Activity：

1. The teacher introduces the meaning of dilation and contraction.
2. The teacher asks students to
   (a) construct a triangle ABC,
   (b) create a reference point O,
   (c) dilate the triangle ABC by a factor 2 with respect to the reference point O,
   (d) label the new triangle as the triangle PQR.

   An example is shown in Figure 1.1.

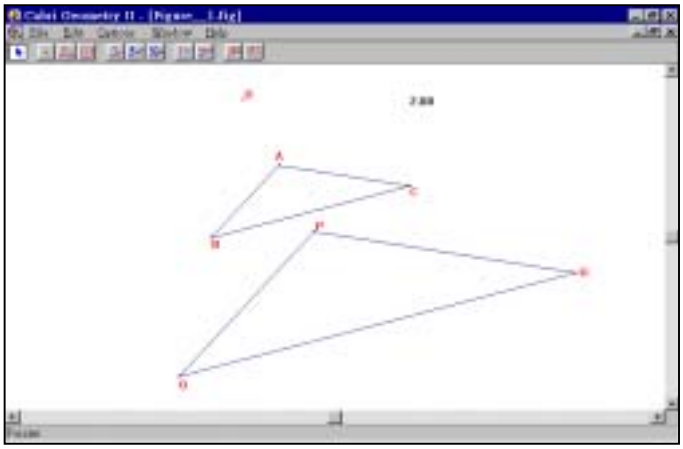

Figure 1.1

3. The teacher asks students to do the investigations on Worksheet 1.1.
   (a) Drag each of the vertices of the triangle ABC and the reference point O. Observe the change. Does the triangle PQR have the same shape as the triangle ABC?
   (b) Measure the lengths, the angles, the perimeters and the areas of the two triangles. What can you say about the changes among these measurements?
   (c) Which property does the triangle ABC preserve under dilation?
   (d) What is the use of the reference point O?
   (e) Is there any relationship between the reference point O and the corresponding vertices of the two triangles (e.g. the points B and Q)? What is the relationship?
   (f) Change the dilation factor to other values such as 0.5, 1.0, 1.5, 3.0 etc. Describe what you observe about the change in the size of the triangle ABC under dilation.
   (g) Try any 2-D figures (e.g. rectangle, trapezium, etc) other than triangles to see the effect of dilation on them.

4. Students are divided into groups. The teacher asks them to discuss their results among their group members.

5. Some of the group representatives are asked to present their findings to the class. The teacher can make comments at appropriate times.

6. The teacher invites volunteers to generalize the results or guides students to reach the conclusion about the effect of dilation and contraction on 2-D figures.

Worksheet 1.1 : Dilation of 2-D Figures

Instruction：

1. Construct a triangle ABC.
2. Create a reference point O.
3. Dilate the triangle ABC by a factor 2 with respect to the reference point O.
4. Label the new triangle as triangle PQR.

Investigation：

1. Drag each of the vertices of the triangle ABC and the reference point O. Observe the change.
   Does the triangle PQR have the same shape as the triangle ABC?

   _____

   _____

   _____

2. Measure the lengths, the angles, the perimeters and the areas of the two triangles. Are there changes among these measurements? What are these changes?

   _____

   _____

   _____

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3. Which property does the triangle ABC preserve under dilation?

   _____

   _____

4. What is the use of the reference point O?

5. Is there any relationship between the reference point O and the corresponding vertices of the two triangles (e.g. the points B and Q)?

6. Change the dilation factor to other values such as 0.5, 1.0, 1.5, 3.0 etc. Describe what you observe about the change in the size of the triangle ABC under dilation.

7. Try any 2-D figures (e.g. rectangle, trapezium, etc) other than triangles to see the effect of dilation on them. Write down any conjecture you can come up with.

Notes for Teachers：

1. At the beginning of the lesson, the teacher introduces the terms ‘reference point’ and ‘dilation factor’ in order to perform the dilation in the activity.

2. Students are expected to discover that if the dilation factor equals 2.0, the area of the image triangle is four times that of the original one. If the dilation factor equals 1.0, the image is coincident with the original figure. If the dilation factor is 0.5, the image is contracted to one quarter of the size of the original one.

3. For less able students, the teacher can give them the following table to record the measurements of the lengths, angles and areas before the investigation.

ΔABC		ΔPQR	
AB		PQ	
BC		QR	
CA		RP	
∠ABC		∠PQR	
∠BCA		∠QRP	
∠CAB		∠RPQ	
Perimeter		Perimeter	
Area		Area	

4. Students can use the **Collinear** tool from the **Check property** toolbox to check that the points O, B and Q are collinear. The dilation must have a reference point for consideration. If we draw three rays from O to A, B and C respectively, we have the following figure (Figure 1.2).

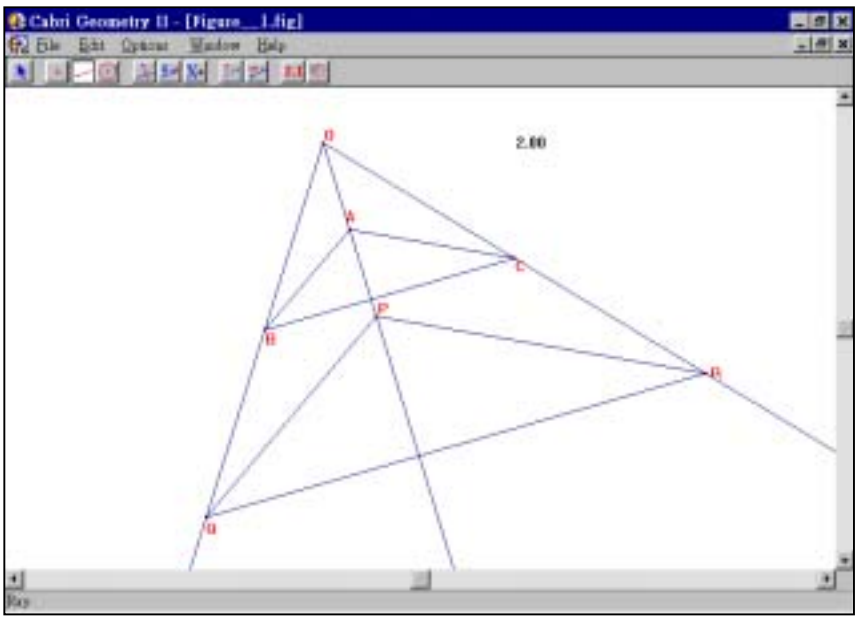


Figure 1.2

5. It is easy for students to use the software to check that the dilation preserves the ratios $\frac{OP}{OA} = \frac{OQ}{OB} = \frac{OR}{OC} = \frac{PQ}{AB} = \frac{QR}{BC} = \frac{PR}{AC} = \frac{2}{1}$.

6. The teacher can guide students to see the difference as shown in Figure 1.3 if the dilation point O lies inside the triangle.

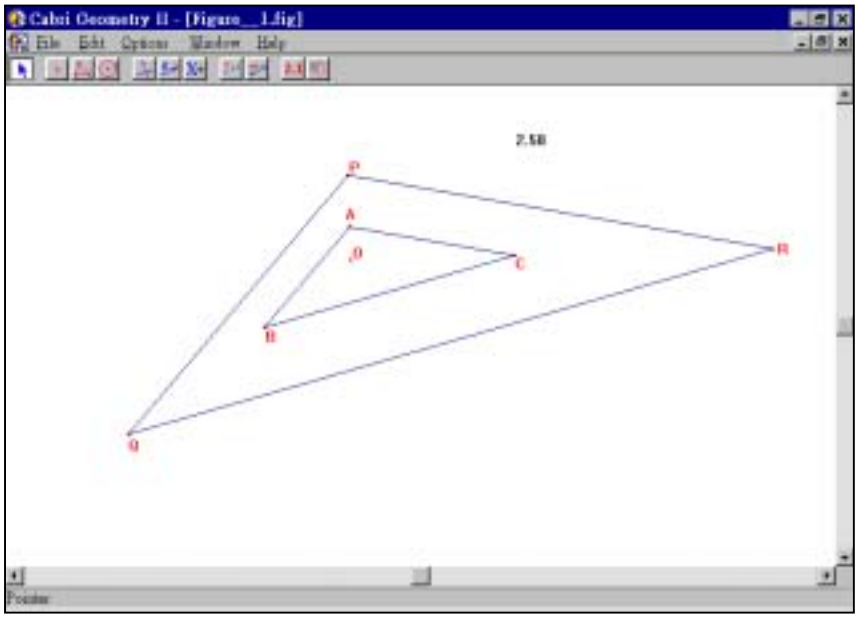


Figure 1.3

7. When students are making dynamic change of the dilation on 2-D figures, they can discover that the factor may be zero or even negative. When the dilation factor is negative, the directions of O to A and O to P are opposite to each other. See Figure 1.4.

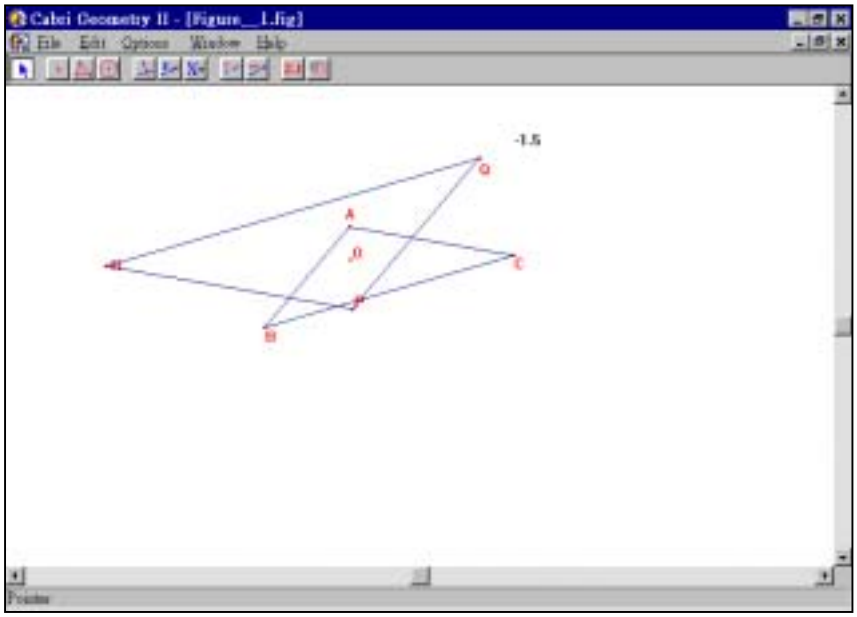
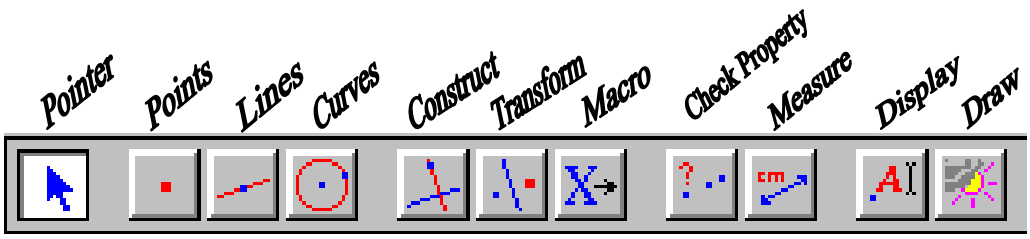


Figure 1.4

Operation Procedure：

(I) Tools in *Cabri Geometry II*

CABRI TOOLBAR



Some of the commonly used icons are listed below:

POINTER

- Pointer
- Rotate
- Dilate
- Rotate and Dilate

POINTS

- Point
- Point on Object
- Intersection Point

LINES

- Line
- Segment
- Ray
- Vector
- Triangle
- Polygon
- Regular Polygon

TRANSFORM

- Translation
- Rotation
- Dilation
- Reflection

CHECK PROPERTY

- Collinear
- Parallel
- Perpendicular
- Equidistant

MEASURE

- Distance and Length
- Area
- Slope
- Angle
- Equation and Coordinates

DISPLAY

- Label
- Comments
- Numerical Edit
- Mark Angle

DRAW

- Hide/Show
- Color
- Fill
- Thick
- Dotted
- Show/Hide Axes
- New Axes
- Define Grid

(II) Construct the triangle ABC

1. Select **Triangle** from the **Lines** toolbox.
2. Construct a triangle in the drawing window.
3. Select **Label** from the **Display** toolbox.
4. Click to select a vertex of the triangle. An edit box appears.
5. Type A in the edit box.
6. Repeat steps 4 and 5 to label the other two vertices as B and C.

(III) Dilate the triangle ABC

1. Select **Numerical Edit** from the **Display** toolbox.
2. Click anywhere in the drawing window. An edit box appears.
3. Type a numerical value 2.0 in the edit box.
4. Press CTRL+U to select **Without units**.
5. Select **Point** from the **Points** toolbox.
6. Move the cursor to any location in the drawing window near the triangle and click once to create a point.
7. Label the point as **O**.
8. Select **Dilation** from the **Transform** toolbox.
9. Select the following in sequence by clicking once: triangle ABC, the point of dilation O and the factor of dilation 2.0. The image triangle is formed.
10. Label the image of the triangle ABC as triangle PQR.

(IV) Measure lengths, angles and areas

1. To measure the distance and length:
   Select **Distance & Length** from the **Measure** toolbox, and then select the two points to be measured.
2. To measure an angle:
   Select **Angle** from the **Measure** toolbox, and then select three points (the vertex and 2 points from the arms of the angle) to specify the angle. The second point must be the vertex.
3. To measure the area:
   Select **Area** from the **Measure** toolbox, and then select the object (the polygon or the circle) of which the area you want to measure.
4. To change the factor of dilation /contraction:
   (a) Double click the numerical value 2.0 to activate the edit box.
   (b) Press (down arrow key) / (up arrow key) in the edit box to change the numerical value.