

## Exemplar



## To Explore the Properties of Parallelogram

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**Objective :** To explore the properties of parallelogram

**Dimension :** Measures, Shape and Space

**Learning Unit :** Quadrilaterals

**Key Stage :** 3

**Materials Required :** *Geometer's Sketchpad*

**Prerequisite Knowledge :** Definition of parallelogram

**Description of the Activity :**

1. The teacher asks students to
  - (a) construct the line segments AB and AC,
  - (b) construct a line through C parallel to AB and a line through B parallel to AC,
  - (c) construct the line segments CD and BD, where D is the point of intersection of the lines in (b),
  - (d) construct diagonals AD and BC and label their point of intersection as E,
  - (e) hide the lines constructed in (b).

Figure 3.1 is an illustration.

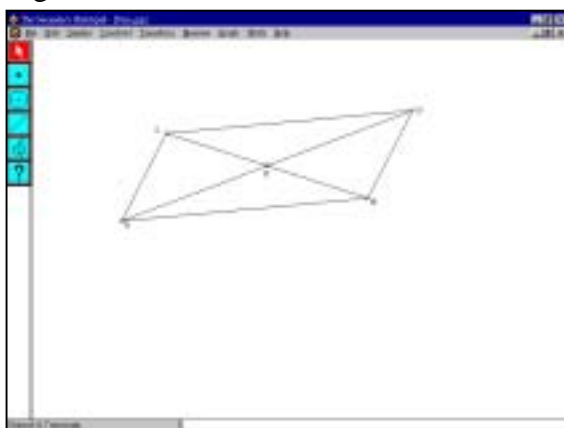


Figure 3.1

- (f) Rename point C as D and point D as C. See Figure 3.2.

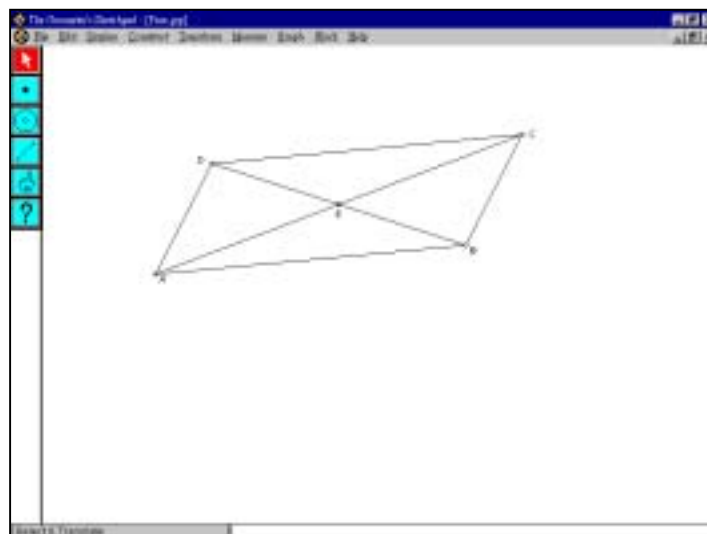


Figure 3.2

2. The teacher asks students to do the investigation in Worksheet 3.1.
  - (a) Measure  $\angle ABC$ ,  $\angle BCD$ ,  $\angle CDA$  and  $\angle DAB$ . What relationships can you observe among these angles?
  - (b) Measure the lengths AB, BC, CD and DA. How are the sides related?
  - (c) Measure the lengths AE, EC, BE and ED. What can you say about the diagonals of a parallelogram?
  - (d) [Optional] What kind of symmetry does a parallelogram have?
3. Students are asked to compare and discuss their results with their classmates. Some students are invited to present their findings to the class. The teacher then helps them summarise the properties of parallelogram.

**Worksheet 3.1: Investigate the Properties of Parallelogram****Instruction :**

1. Construct the line segments AB and AC.
2. Construct a line through C parallel to AB and a line through B parallel to AC.
3. Construct the line segments CD and BD, where D is the point of intersection of the lines mentioned in step 2.
4. Construct diagonals AD and BC and their point of intersection, E.
5. Hide the lines constructed in step 2.
6. Rename point C as D and point D as C.

**Investigation :**

During the investigation, you can drag the vertices of the parallelogram to change its shape.

- (a) Measure  $\angle ABC$ ,  $\angle BCD$ ,  $\angle CDA$  and  $\angle DAB$ .

What relationships can you say about

- (i) the opposite angles of a parallelogram?
- (ii) any two consecutive interior angles (e.g.  $\angle ABC$  and  $\angle BCD$ ) of a parallelogram?

Do the relationships hold for all parallelograms?

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- (b) Measure the lengths AB, BC, CD and DA. How are the sides related? Does it hold for all parallelograms?

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- (c) Measure AE, EC, BE and ED. What can you say about the diagonals of a parallelogram? Does it hold for all parallelograms?

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- (d) [Optional] What kind of symmetry does a parallelogram have?




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**Notes for Teachers :**

1. There are many different methods to construct the parallelogram. The method given here is only a suggestion. The teacher can use other methods to obtain the same figure. For example, we can use translation to construct the opposite sides of a parallelogram. Nevertheless, the level of difficulty and the prerequisite knowledge of students should be considered.
2. Suggested answers to Worksheet 3.1.
  - (a) (i) Opposite angles are equal.
    - (ii) Any two consecutive interior angles are supplementary, i.e., the sum of the two consecutive interior angles is  $180^\circ$ .
  - (b) Opposite sides are equal.
  - (c) Diagonals bisect each other.
  - (d) A parallelogram has  $180^\circ$  rotational symmetry.
3. As the students are assumed to be familiar with the definition of parallelogram, the teacher should ask them to focus upon the properties of the diagonals and the relationship of the angles. At the end of the lesson, the teacher can conclude with a list of all the properties of parallelogram that students have discovered on the blackboard.
4. The teacher should explain to students that the renaming action in step 6 of Worksheet 3.1 is necessary because *Geometer's sketchpad* automatically assign a name to the point constructed in sequence.


**Operation Procedure :****(I) Construct the parallelogram ABCD**

1. Click the **Segment** tool  and drag to draw a horizontal segment **AB**.
2. Click on point **A**, and drag to construct **AC**.
3. Click the **Selection Arrow** tool . Hold down the **Shift** key. Select point **C** and **AB**. Go to the **Construct** menu and choose **Parallel Line**.
4. Hold down the **Shift** key. Select point **B** and **AC**. Select **Construct** | **Parallel Line**.
5. Hold down the **Shift** key. Select the two lines constructed in steps 3 and 4. Select **Construct** | **Point At Intersection**.
6. Hold down the **Shift** key. Select points **C** and **D**. Select **Construct** | **Segment**. Construct line segment **BD** in a similar way.
7. Hold down the **Shift** key. Select points **A** and **D**. Select **Construct** | **Segment**. Construct diagonal **BC** in a similar way.
8. Hold down the **Shift** key. Select the line passing through **B** and **D** and the line passing through **C** and **D**. Select **Display** | **Hide Lines**.
9. Click the **Text** tool  and label the vertices. Double click the label, a **Relabel** window will appear. Enter the new label in the label box.


**(II) Measure the sizes of angles**

1. To measure  $\angle ABC$ , click the **Selection Arrow** tool .
2. Hold down the **Shift** key. Select the points **A**, **B** and **C** in order.
3. Select **Measure** | **Angle**.
4. Repeat steps 1 to 3 to measure the other angles.

**(III) Measure the lengths of line segments**

1. To measure the length of **AB**, click the **Selection Arrow** tool  and click the line segment **AB** to select it.
2. Select **Measure** | **Length**.
3. Repeat steps 1 and 2 to measure the lengths of the other line segments.

**(IV) Create a “Fast” button for construction**

1. Use a selection marquee to select your constructed parallelogram.
2. Select **Work | Make Script**.
3. Select **File | New Sketch**. Use the **Point** tool  to construct three points in your sketch.
4. Select all the points, click back onto your **Script** window, and click on the **Fast** button. You should see a parallelogram constructed in your sketch.
5. Click back on the **Script** window. Select **File | Save**. Any time in the future you need such a figure, simply open your **script** and then construct three points. Click the **Fast** button and you can get the figure.

(Similar steps could be applied for sketching a square, rectangle or rhombus)