Exemplar 

Solving Simultaneous Linear Equations Graphically

Objective： To solve simultaneous linear equations by using a graphing software

Dimension： Number and Algebra

Learning Unit： Linear Equations in Two Unknowns

Key Stage： 3

Material Required： Graphing software - *Graphmatica*

Prerequisite Knowledge：
(1) Graphical method in solving simultaneous linear equations by using paper and pencil
(2) Rounding off numbers to the required number of decimal places

Description of the Activity：

1. The teacher gives a brief revision on the graphical method of solving simultaneous linear equations in two unknowns by using paper and pencil.

2. The teacher demonstrates how to use *Graphmatica* by using the example in Part I of Worksheet 6.1 :

   Solve $\begin{cases} x-2y=1 \\ 2x+3y=12 \end{cases}$ graphically.

   The students follow the working steps of the teacher during the demonstration as a practice. The teacher should point out that *Graphmatica* does not provide accurate solutions but provides opportunities to find the solutions up to a certain degree of accuracy.

3. After drawing the graphs of $x - 2y = 1$ and $2x + 3y = 12$, the teacher asks students to find the solution of the equations correct to one decimal place by using Figure 6.1. As the horizontal and vertical gridlines are not dense enough, it is quite difficult for them to find the accurate solution. The teacher can use the **Zoom in** function of *Graphmatica* until a suitable level of accuracy to find the solution. See Figure 6.2.

4. Students are asked to work in pairs to finish Part II of Worksheet 6.1 by using *Graphmatica*.

5. After students have completed Part II, the teacher can give them the answers for checking.

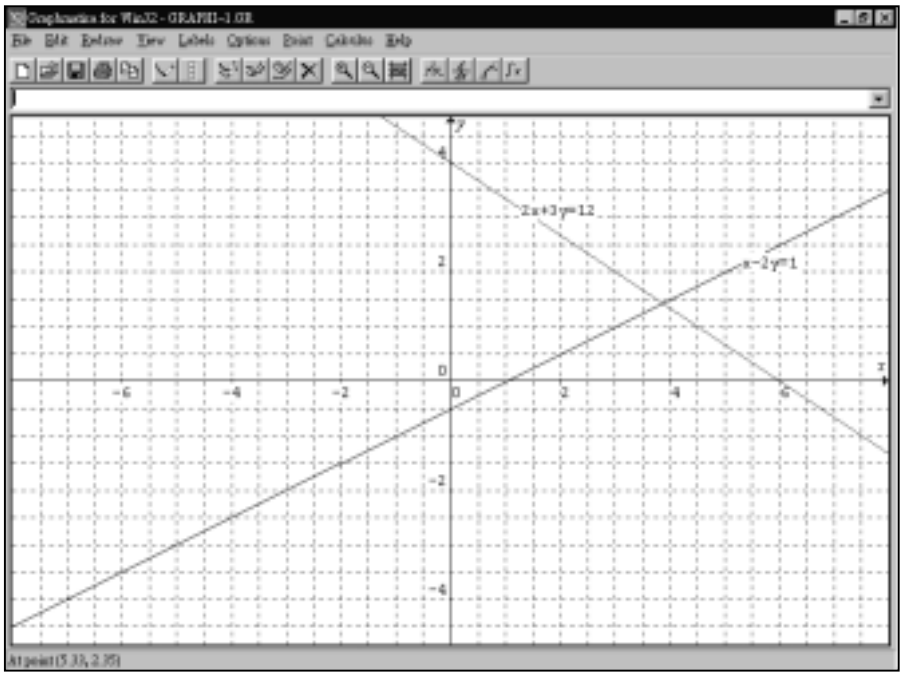

Figure 6.1

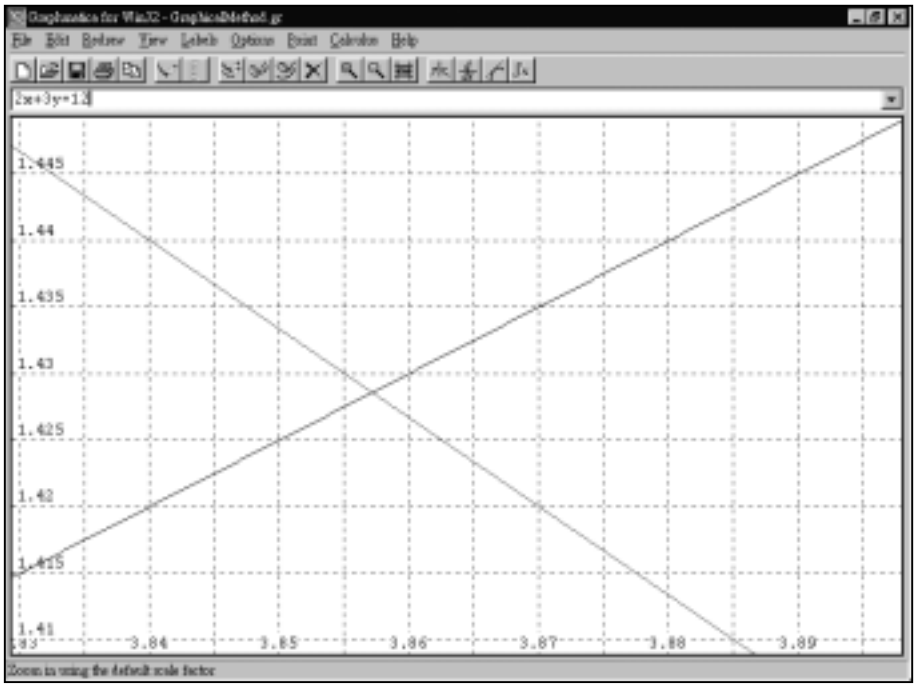

Figure 6.2

Worksheet 6.1: Use Graphmatica to Solve Simultaneous Linear Equations

Part I : Example

Use *Graphmatica* to solve the simultaneous linear equations graphically

$$(*)\ \begin{cases} x - 2y = 1 \\ 2x + 3y = 12 \end{cases}.$$

1. Draw the graphs of $x - 2y = 1$ and $2x + 3y = 12$ and label them as shown in Figure 6.1a.

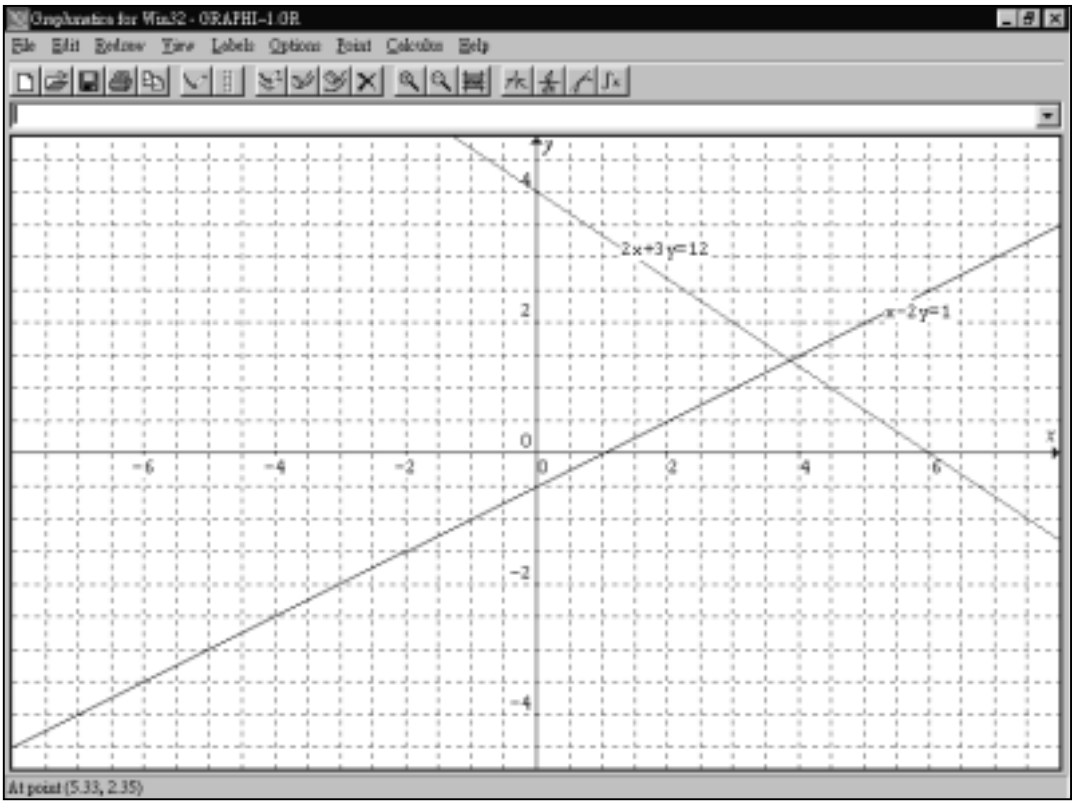

Figure 6.1a

2. From the above graph, is it easy to determine the solution of $(*)$ correct to one decimal place? Why?

_____

_____

_____

_____

3. Use the **Zoom in** function to magnify the graph so that you can determine the solution of (∗) correct to one decimal place. Print out your zoomed graph and stick it below.

From the above graph, we can see that the solution to (∗) is

x = _____ , y = _____ , correct to one decimal place.

4. Continue to zoom in so that you can get the solution of (∗) correct to 2 decimal places. Print out your zoomed graph and stick it below.

From the above graph, we can see that the solution to (∗) is

$x =$ _____ , $y =$ _____ , correct to 2 decimal places.

Part II Exercise

Use *Graphmatica* to solve the following simultaneous linear equations. If the answer is not exact, correct your answers to 2 decimal places. Print out your zoomed graphs and stick them on a paper. Also write down your solutions.

1. $\begin{cases} y = 3x \\ x - 2y + 6 = 0 \end{cases}$

2. $\begin{cases} 2x + 5y = 6 \\ 3x - y + 6 = 0 \end{cases}$

3. $\begin{cases} 2x - 3y = 12 \\ x - 5y + 5 = 0 \end{cases}$

4. $\begin{cases} x - 2y - 4 = 0 \\ 5x - 3y + 10 = 0 \end{cases}$

Notes for Teachers :

1. Answers for Part I of Worksheet 6.1.

   Point 2 It is not easy to determine the solution to one decimal place as both of the horizontal and vertical gridlines are not dense enough.

   Point 3 The solution for (*) is $x = 3.9$, $y = 1.4$, correct to 1 decimal place.

   Point 4 The solution for (*) is $x = 3.86$, $y = 1.43$, correct to 2 decimal places.

2. Answers for Part II of Worksheet 6.1.

Question	Answer
1	$x = 1.2, \quad y = 3.6$
2	$x = -1.41$, $y = 1.76$ (correct to 2 decimal places)
3	$x = 10.71$, $y = 3.14$ (correct to 2 decimal places)
4	$x = -4.57$, $y = -4.29$ (correct to 2 decimal places)

3. For Figure 6.2, if the solution of the equations is (x_o,y_o), we can see from the graph that $3.855 < x_o < 3.86$ and $1.425 < y_o < 1.43$. We can then conclude that $x_o=3.86$ and $y_o=1.43$, correct to 2 decimal places. The teacher can go on to discuss with students that for the above ranges of x_0 and y_0, it is not possible to find the answers correct to 3 decimal places.

4. For less able students, the teacher can provide some more exercises on the rounding off of numbers before Part II of Worksheet 6.1.

5. For abler students, they can use this worksheet as a self-learning exercise by following the Operation Procedure on their own.

Operation Procedure :

Using *Graphmatica* to solve the simultaneous linear equations
$\begin{cases} x-2y=1 \\ 2x+3y=12 \end{cases}$ graphically.

1. The demo version of *Graphmatica* can be downloaded from the web site http://www8.pair.com/ksoft/ .

2. After launching the program *Graphmatica*, input the equation $x - 2y = 1$ in the dialogue box and then press **Enter**. A line will be plotted.

3. Plot another graph $2x + 3y = 12$ in a similar way.

4. To change the colour of the graph, select **View | Colors**. In the dialogue box, select the colour preferred.

5. To label the graph of the equation $x - 2y = 1$, select **Labels | Annotate**. In the dialogue box, enter the equation '$x - 2y = 1$' and press **Place**. Go to the desired location on the graph and click the mouse to label it. Repeat the same procedure for the graph of the equation $2x + 3y = 12$.

6. To magnify the graph, use the **Zoom in** function. Move the mouse to a point near the upper-left corner of the point of intersection of the two graphs. Drag it to select a rectangular region for zooming. See Figure 6.3 below.

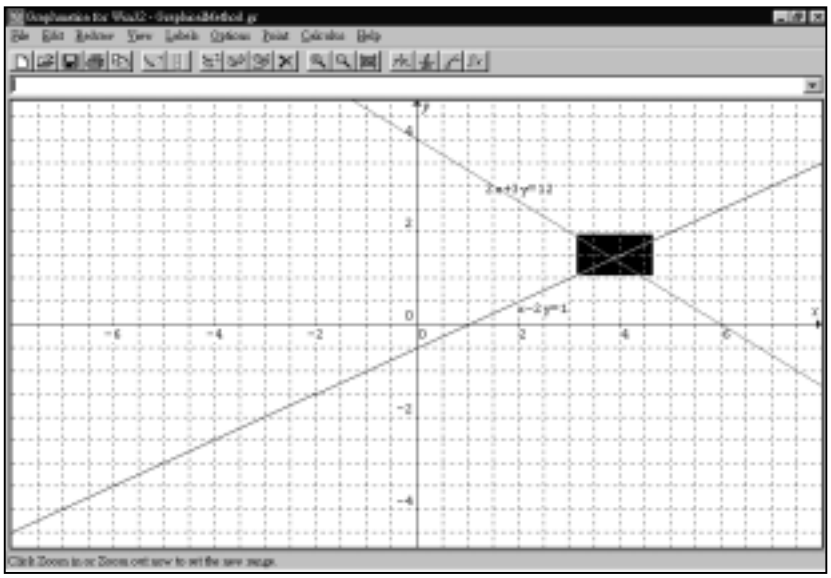

Figure 6.3

To zoom in, click the **Zoom in** icon to get the result shown in Figure 6.4 below.

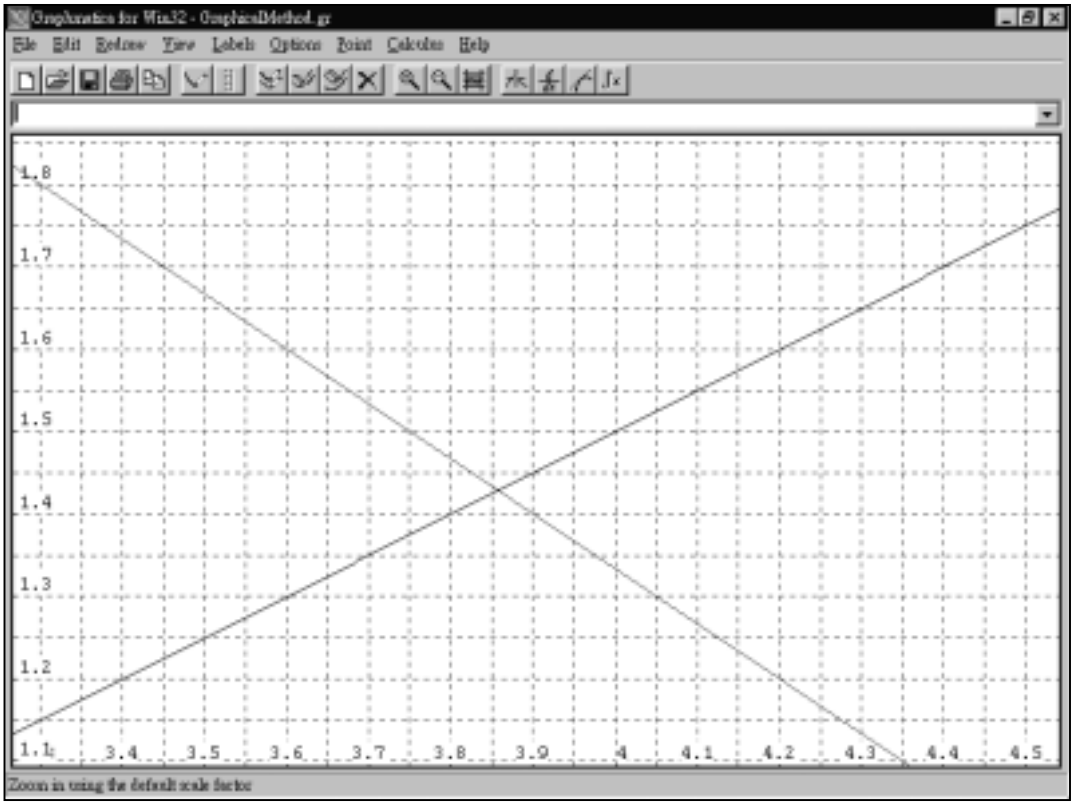

Figure 6.4

7. To obtain the solution correct to any decimal places, zoom in until you get the sufficient level of accuracy.
8. Click the **Default grid** icon to go back to the default size of the graph.
9. To create a scroll bar for the graph, select **Options | Show Scrollbars**.