Exemplar 

Number of Solutions of Simultaneous Linear Equations

Objective： To explore the number of solutions of simultaneous linear equations

Dimension： Number and Algebra

Learning Unit： Linear Equations in Two Unknown

Key Stage： 3

Materials Required： *Excel* and the file 10_ex_e.xls

Prerequisite Knowledge：
(1) Graphs of linear equations in two unknowns
(2) Solving simultaneous linear equations by the algebraic method and the graphical method

Description of the Activity：

1. The teacher gives a brief revision on the methods of solving simultaneous linear equations.

2. Students are divided into groups.

3. The teacher distributes Worksheet 10.1 to students.

4. Students are asked to solve Question 1 in Worksheet 10.1 by the algebraic method.

5. After completing Question 1, students open the *Excel* file 10_ex_e.xls to do Questions 2 to 7. A Visual Basic Application programme is contained in the file. The programme enables students to see the interactive change on the graphs when clicking the built-in buttons. See Figure 10.1.

6. Some group representatives are invited to present their findings or conclusions to the class. The teacher can give comments and summarize the conclusions.

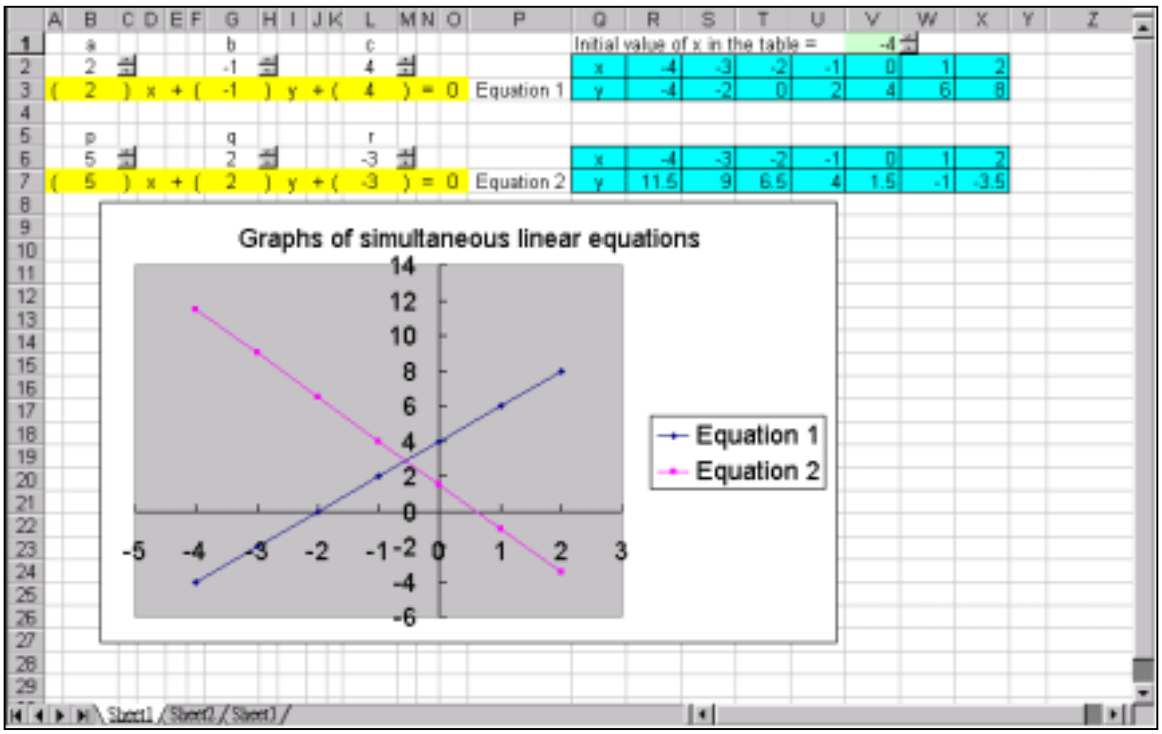

Initial value of x in the table = -4
a 2
b -1
c 4
(2) x + (-1) y + (4) = 0 Equation 1
p 5
q 2
r -3
(5) x + (2) y + (-3) = 0 Equation 2
x -4 -3 -2 -1 0 1 2
y -4 -2 0 2 4 6 8
x -4 -3 -2 -1 0 1 2
y 11.5 9 6.5 4 1.5 -1 -3.5
Graphs of simultaneous linear equations
Equation 1
Equation 2
Sheet1 Sheet2 Sheet3

Figure 10.1

Worksheet 10.1 : Number of Solutions of Simultaneous Linear Equations

Part I

1. Solve the following simultaneous linear equations below by the algebraic method.

$$\begin{cases} 3x - y = 0 \\ 2x - 3y + 7 = 0 \end{cases}$$

2. Open the *Excel* file 10_ex_e.xls.

The *Excel* file provides a program to solve the simultaneous equations

$$\begin{cases} ax + by + c = 0 \\ px + qy + r = 0 \end{cases}$$ graphically,

where a, b, c, p, q and r are real numbers.

Solve the simultaneous linear equations in question 1 again by inputing the corresponding values of a, b, c, p ,q and r in the cells B2, G2, L2, B6, G6 and L6 respectively.

You can read the solution either from the graphs or from the tables.

Answer:

(a) The number of the point(s) of intersection of the graphs is _____.

(b) The number of solution(s) is _____.

(c) The solution is (,).

3. Use the *Excel* file to find the number of solution(s) and point(s) of intersection of the following simultaneous linear equations in Table 10.1.

(a)

Equation 1 $ax + by + c = 0$	**Equation 2** $px + qy + r = 0$	**Number of solution(s)**	**Number of point(s) of intersection**	$\frac{a}{p}$	$\frac{b}{q}$
$3x - y = 0$	$2x - 3y + 7 = 0$				
$3x - y + 1 = 0$	$2x - 3y + 7 = 0$				
$3x - y + 2 = 0$	$2x - 3y + 7 = 0$				
$3x - y + 3 = 0$	$2x - 3y + 7 = 0$				
$3x - y + 4 = 0$	$2x - 3y + 7 = 0$				
$3x - y + 5 = 0$	$2x - 3y + 7 = 0$				
$3x - y + 6 = 0$	$2x - 3y + 7 = 0$				
$3x - y + 7 = 0$	$2x - 3y + 7 = 0$				
$3x - y + 8 = 0$	$2x - 3y + 7 = 0$				

Table 10.1

You can change the value of c by entering your value in the cell L2 directly or clicking the **SpinUp** or **SpinDown** buttons in the cell M2.
Clicking the **SpinUp** or **SpinDown** buttons in the cells W1 can change the initial value of x in the tables.

(b) What happens to the graph of Equation 1 when you click the **SpinButton** in the cell M2 to change the value of c?

_____

_____

(c) From the results in Table 10.1, is there any relation between a/p and b/q?

_____

_____

(d) Now choose some other values for a, b and c. Input the values you choose in the *Excel* file. For each value of a and b, click the **SpinButton**  of c to change it to different values.

Observe the effect of different values of c on the number of solutions. Write down your conclusion below. Discuss with your group members.

_____

_____

_____

_____

Part II

4. Input values of a, b, c, p, q and r in the *Excel* file according to the simultaneous linear equations: $\begin{cases} 2x - y + 4 = 0 \\ -4x + 2y - 3 = 0 \end{cases}$.

   (a) Do the graphs of the two equations intersect? _____

   (b) How many solution(s) satisfy the equations? _____

   (c) What is the relation between these two graphs? _____

   (d) What is the relation between the number of intersecting point and the number of solution?

   _____

   _____

   You can alter the initial value of *x* by clicking the **SpinButton**  in the cell W1 to see whether the two graphs intersect or not.

5. In some cases you will observe that there are no points of intersection for the graphs and therefore, no solutions for the simultaneous linear equations.

   (a) Some of the equations in Table 10.2 are omitted. You are now required to construct suitable equations so that there are no solutions for the simultaneous linear equations in each question. Write your answers in the table.

Question	**Equation 1** $ax + by + c = 0$	**Equation2** $px + qy + r = 0$	$\frac{a}{p}$	$\frac{b}{q}$	$\frac{c}{r}$
1	$x + 2y - 3 = 0$				
2	$3x + y + 1 = 0$				
3	$x - 2y = 0$				
4		$4x+3y - 5 = 0$			
5		$-2x+y + 10 = 0$			
6		$x+y+3 = 0$			

Table 10.2

(b) Given that the two equations $\begin{cases} ax + by + c = 0 \\ px + qy + r = 0 \end{cases}$ have no solutions, write down the relations, if any, between a, b, c ,p ,q and r with reference to the results in Table 10.2.

_____

_____

Part III

6. Given two simultaneous linear equations $\begin{cases} 2x - y + 4 = 0 \\ -4x + 2y - 3 = 0 \end{cases}$, input the corresponding values of a, b, c, p ,q and r in the *Excel* file.
Now choose -3 as the value of r for the second equation.

(a) Try to change the value of r by clicking the **SpinButton**  of r so that the two graphs coincide.
What is the value of r? _____

(b) Can you suggest some possible solutions from the tables in the *Excel* file? Write down some of the solutions here:
(,), (,), (,) , (,).

(c) From the above results, can you draw any conclusion? Write it down.

_____

_____

(d) What are the relations between a, b, c, p, q and r in this case when the two graphs of the equations $ax + by + c = 0$ and $px + qy + r = 0$ coincide.

_____

_____

Part IV

Conclusion

7. Summarize all your findings from Part I, II and III and write your conclusions below.

Given two equations $\begin{cases} ax + by + c = 0 \\ px + qy + r = 0 \end{cases}$.

Number of point(s) of intersection	Number of solution(s)	Relations between a, b, c ,p ,q and r
0		
1		
Infinitely many		

Notes for Teachers:

1. The objective of the worksheet is to explore the number of solutions of simultaneous linear equations rather than to find the solution(s). The graphs just illustrate whether there is a point of intersection. Part I focuses on two non-parallel lines, Part II on two parallel lines and Part III on two overlapping lines.

2. From the graphs it can be observed easily that the number of points of intersection can be 0, 1 or infinitely many. Students may use more than one solution to describe the infinite case. The teacher may introduce the term "infinitely many solutions", "one solution" and "no solution" to students. The terms "consistent" and "inconsistent" may also be introduced for abler students.

3. In the *Excel* file, if we set the value of b to zero in the cell G2, we will have problems in plotting the graph because in the table, the value of y is calculated from the formula $y = \frac{-(ax + c)}{b}$. y is only well-defined when b is not equal to zero. The teacher may point out this special case to students. In this case, it is a vertical line.

4. Suggested answers for Worksheet 10.1:

Question	**Answer**						
1	(1,3)						
2	(a) 1 (b) 1 (c) (1,3)						
3	(a)	Equation 1 $ax + by + c = 0$	Equation 2 $px + qy + r = 0$	Number of solution(s)	Number of point(s) of intersection	$\frac{a}{p}$	$\frac{b}{q}$
		$3x - y = 0$	$2x - 3y + 7 = 0$	1	1	$\frac{3}{2}$	$\frac{1}{3}$
		$3x - y + 1 = 0$	$2x - 3y + 7 = 0$	1	1	$\frac{3}{2}$	$\frac{1}{3}$
		$3x - y + 2 = 0$	$2x - 3y + 7 = 0$	1	1	$\frac{3}{2}$	$\frac{1}{3}$
		$3x - y + 3 = 0$	$2x - 3y + 7 = 0$	1	1	$\frac{3}{2}$	$\frac{1}{3}$
		$3x - y + 4 = 0$	$2x - 3y + 7 = 0$	1	1	$\frac{3}{2}$	$\frac{1}{3}$
		$3x - y + 5 = 0$	$2x - 3y + 7 = 0$	1	1	$\frac{3}{2}$	$\frac{1}{3}$
		$3x - y + 6 = 0$	$2x - 3y + 7 = 0$	1	1	$\frac{3}{2}$	$\frac{1}{3}$
		$3x - y + 7 = 0$	$2x - 3y + 7 = 0$	1	1	$\frac{3}{2}$	$\frac{1}{3}$
		$3x - y + 8 = 0$	$2x - 3y + 7 = 0$	1	1	$\frac{3}{2}$	$\frac{1}{3}$

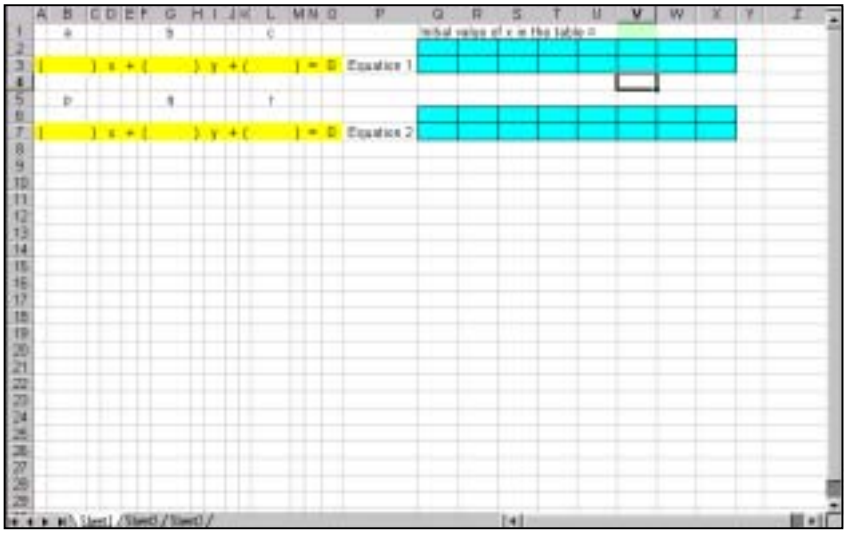
<table>
<tr><td>3</td><td>(b) The graph shifts upwards or downwards and parallel to the original line.
(c) $\frac{a}{p}$ does not equal to $\frac{b}{q}$.
(d) The change in the values of c will have no effects on the number of solution(s) if the original equations have only one solution.</td></tr>
<tr><td>4</td><td>(a) No
(b) 0
(c) Parallel to each other.
(d) The same</td></tr>
<tr><td>5</td><td>(a)
<table>
<tr><th>Qu</th><th>Equation 1
$ax + by + c = 0$</th><th>Equation 2
$px + qy + r = 0$</th><th>$\frac{a}{p}$</th><th>$\frac{b}{q}$</th><th>$\frac{c}{r}$</th></tr>
<tr><td>1</td><td>$x+2y-3=0$</td><td>$x+2y=0$</td><td>1</td><td>1</td><td>$-\frac{3}{2}$</td></tr>
<tr><td>2</td><td>$3x+y+1=0$</td><td>$6x+2y+3=0$</td><td>$\frac{1}{2}$</td><td>$\frac{1}{2}$</td><td>$\frac{1}{3}$</td></tr>
<tr><td>3</td><td>$x-2y=0$</td><td>$x-2y+5=0$</td><td>1</td><td>1</td><td>0</td></tr>
<tr><td>4</td><td>$4x+3y+2=0$</td><td>$4x+3y-5=0$</td><td>1</td><td>1</td><td>$-\frac{2}{5}$</td></tr>
<tr><td>5</td><td>$2x-y+10=0$</td><td>$-2x+y+10=0$</td><td>-1</td><td>-1</td><td>1</td></tr>
<tr><td>6</td><td>$-3x-3y+1=0$</td><td>$x+y+3=0$</td><td>-3</td><td>-3</td><td>$\frac{1}{3}$</td></tr>
</table>
(b) $\frac{a}{p} = \frac{b}{q} \neq \frac{c}{r}$</td></tr>
<tr><td>6</td><td>(a) –8
(b) (-2,0), (-1,2), (0,4), (1,6) , etc.
(c) When two lines overlap, there are infinitely many solutions.
(d) $\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$</td></tr>
<tr><td>7</td><td>
<table>
<tr><th>Number of point(s) of intersection</th><th>Number of solution(s)</th><th>Relations between a, b, c, p, q, and r</th></tr>
<tr><td>0</td><td>0</td><td>$\frac{a}{p} = \frac{b}{q} \neq \frac{c}{r}$</td></tr>
<tr><td>1</td><td>1</td><td>$\frac{a}{p} \neq \frac{b}{q}$</td></tr>
<tr><td>infinitely many</td><td>infinitely many</td><td>$\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$</td></tr>
</table>
</td></tr>
</table>

Note: The answers in bold to 5(a) are not unique.

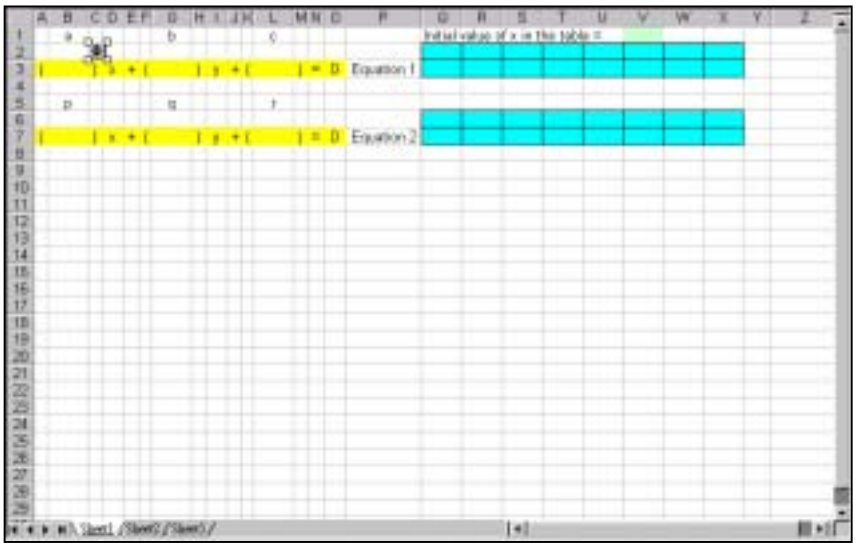
Operation Procedure :

(I) Create the SpinUp and SpinDown button to change the value of a cell

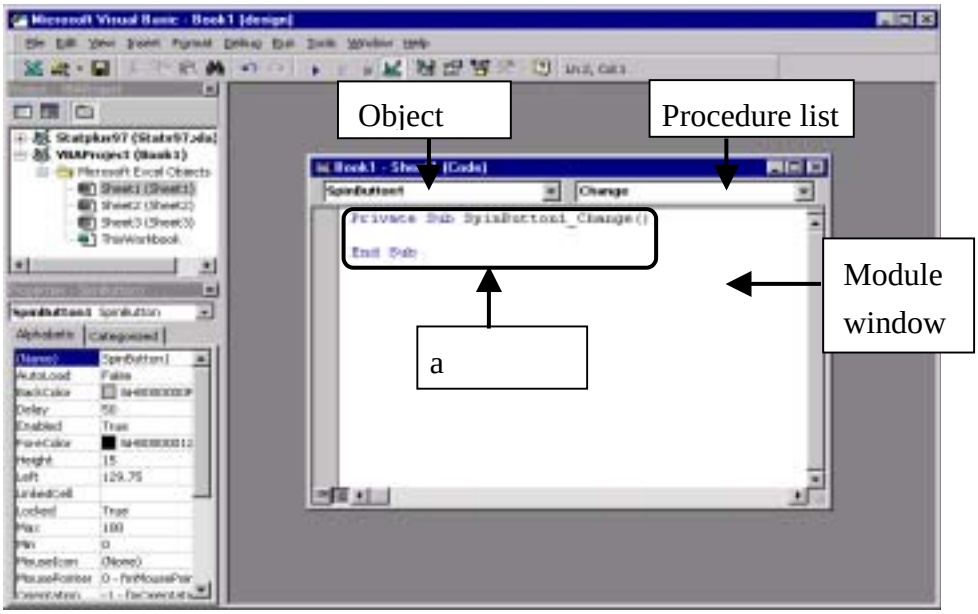
1. Open a new *Excel* file.
2. Construct the worksheet in the format as shown in the following figure.
3. Choose a suitable column width for each column.



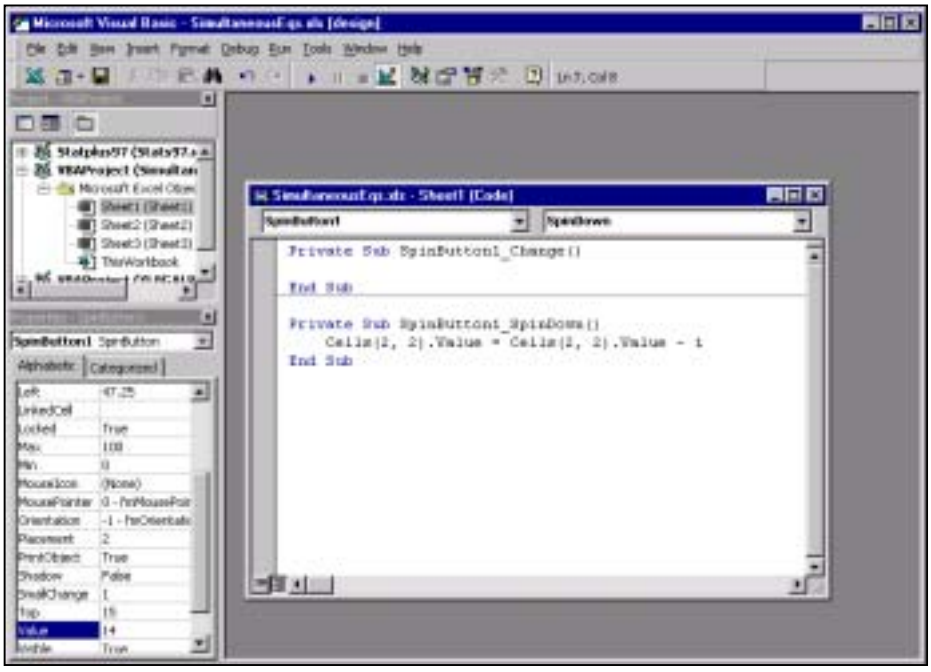
4. Select the **View | Toolbars | Control** Toolbox from the pull-down menu.
5. Click to select the **SpinButton**  icon. Click on the cell C2 to place the **SpinButton**. See the following figure.



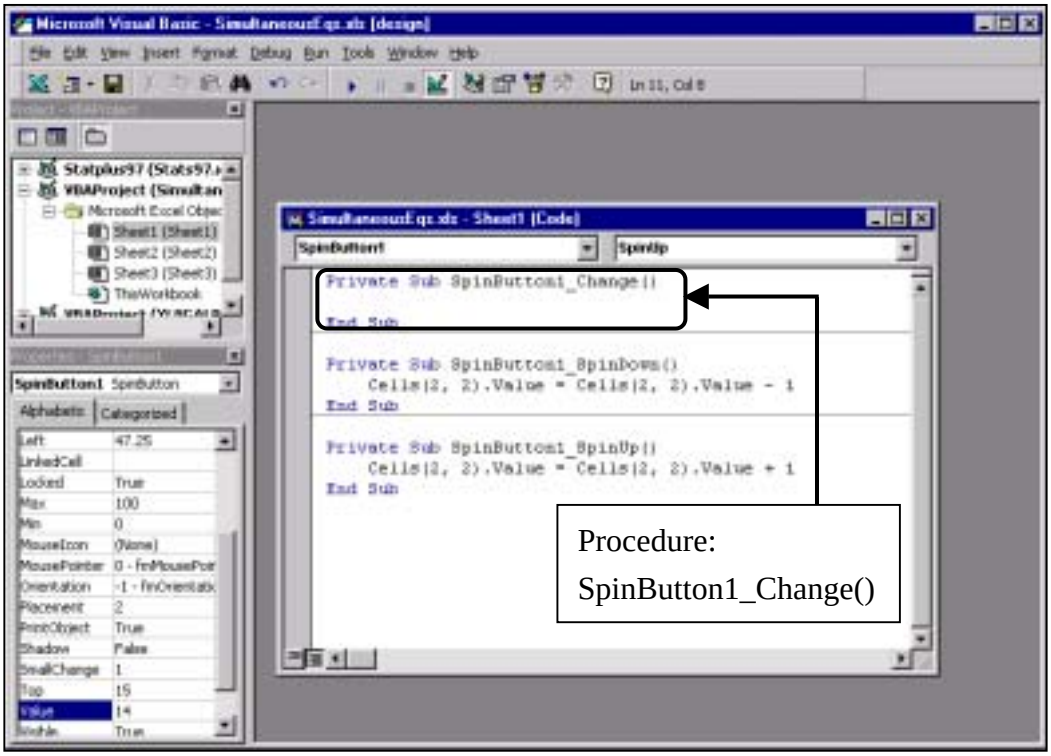
6. Double click the **SpinBotton** to switch to the Visual Basic Edition. In the Code Window, there are the Object list, Procedure list, Module window and Procedure (see the following figure).



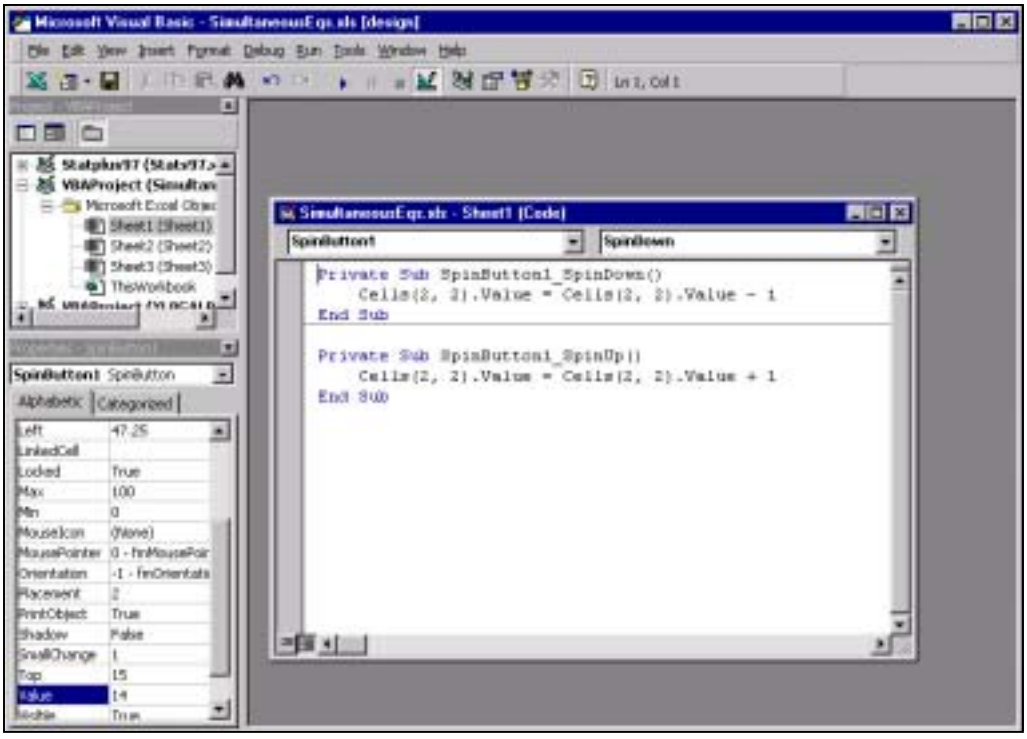
7. Click the **Procedure List** to choose the option **SpinDown**.
Type "Cells(2,2).Value = Cells(2,2).Value – 1" in the 2nd row of the **SpinDown** Procedure to activate the **SpinDown** process as shown in the figure below. Each time when the **SpinDown** button is clicked, the number in the cell B2 will be decreased by 1. Note that "Cells(i,j)" refers to the cell in the i-th row and j-th column.



8. Click the **Procedure List** to choose the option **SpinUp**.
Enter "Cells(2,2).Value = Cells(2,2).Value + 1" in the 2nd row of the **SpinUp** Procedure to activate the **SpinUp** process as shown in the figure below.



9. Highlight the Procedure "Private Sub SpinButton1_Change( )" and "End Sub". Delete these two lines. The result is shown in the following figure.



10. Unselect the icon of design mode  on the tool bar. Click the *Excel* icon  on the tool bar to go back to the Excel worksheet. Click the **SpinUp** button  and the **SpinDown** button  , the value in the cell B2 will change.
11. Add SpinButtons in the cells H2, M2, C6, H6, M6 and W1 in similar ways.

(II) Create the table and plot the graph of a linear equation

1. Enter the formula "=V1" the cell R2 to copy the initial value of *x* in the cell V1 to the cell R2.
2. Enter the formula "=R2+1" in the cell S2 to add a value 1 to the cell R2.
3. Select the cell S2. Go to the lower right corner of the cell S2. Drag the fill handle of cell S2 to X2 in order to copy the formula in the cell S2 to cells from T2 to X2.
4. Enter the formula "=($B$2*R2+$L$2)*(-1)/$G$2" in the cell R3 for the formula $y = \frac{(ax + c) \times (-1)}{b}$ and press **Enter**. Select the cell R3. Drag the fill handle of cell R3 to X3 in order to copy the formula in the cell R3 to cells from S3 to X3.
5. To draw the graphs of simultaneous linear equations, choose the Chart Wizard icon  and select the Chart type "XY(Scatter)" and Chart Sub-type "Scatter" with data points connected by lines. Follow the steps guided by the dialogue boxes until Finish.

(III) Protect the worksheet

1. Open the above *Excel* file.
2. Select the cell B2. Right click to choose **Format Cells**. Select the tab **Protection.** Uncheck the box of **Locked** and press **OK**. Unlock the cells G2, L2, B6, G6, L6 and V1 in the same way.
3. In the Pulldown menu, choose **Tools | Protection | ProtectSheet**. Enter password if you want. Press **OK**. The worksheet is then protected. You can only change the values of the above unchecked cells only.
4. To unprotect the worksheet, select **Tools | Protection | Unprotect Sheet** from the Pulldown menu. Enter the password if required.