



## Exemplar

## Pythagoras' Theorem

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**Objective :** To recognize and appreciate different proofs of Pythagoras' Theorem

**Dimension :** Measures, Shape and Space

**Learning Unit :** Pythagoras' Theorem

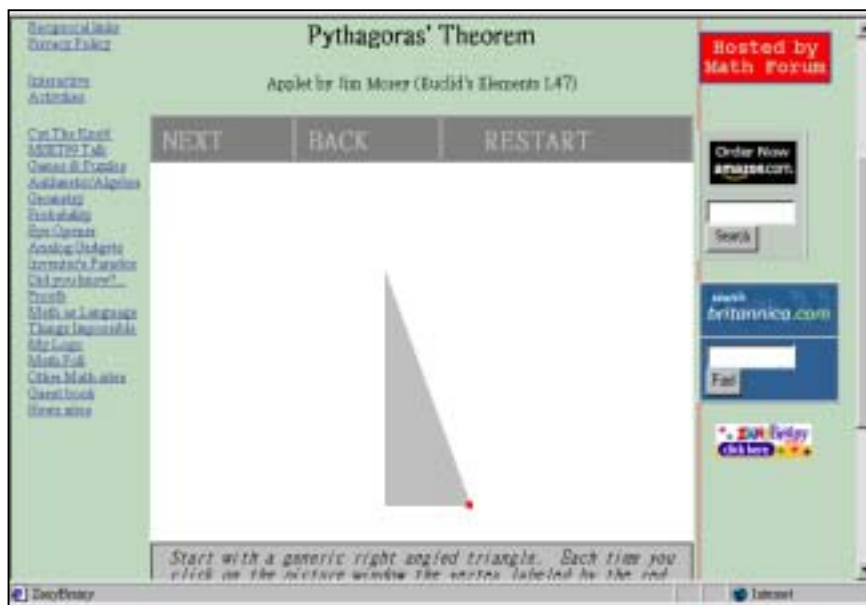
**Key Stage :** 3

**Material Required :** *Internet*

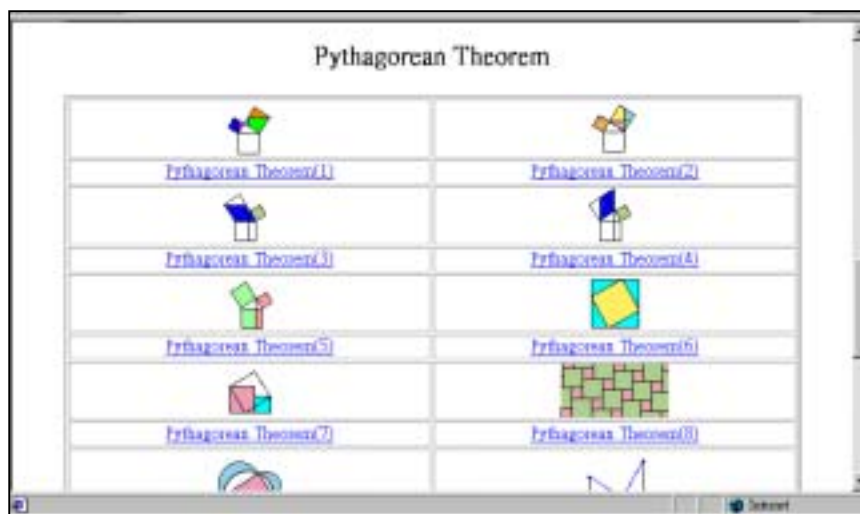
**Prerequisite Knowledge :** Manipulation of simple polynomials

**Description of the Activity :**

1. The teacher groups students in pairs and distributes Worksheet 11.1 to each group.
2. Students complete Part I “Discovering Pythagoras’ Theorem” in the worksheet by using the data obtained from the web site provided.
3. Some students are invited to present their conjectures to the whole class.
4. In Part II “Proof of Pythagoras’ Theorem”, students visit the web site <http://www.cut-the-knot.com/pythagoras/morey.html> to observe an interactive proof for the Pythagoras’ Theorem. The teacher can review each step in the proof with students to make sure that students can actually follow the proof.



5. Students go to the web site <http://www.ies.co.jp/math/products/geo2/menu.html> to try the interactive proof.



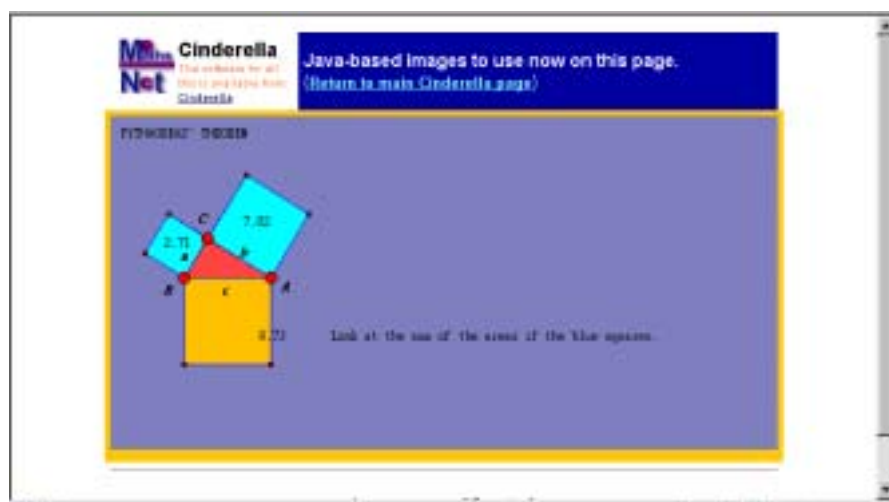
6. The teacher gives the reference web sites provided in the worksheet to students and asks them to find out two other proofs of the Pythagoras' Theorem from these web sites. Students have to write down the proof in their own words.
7. Some students are invited to present their proofs to the whole class.
8. The teacher gives comments when appropriate.

## Worksheet 11.1: Pythagoras' Theorem and Its' Proofs

### Part I : Discovering Pythagoras' Theorem

1. Go to the web site

<http://www.mathsnet.net/dynamic/cindy/pythag.html>.



2. You can find a right-angled triangle ABC with right angle at C. Three squares are built on the three sides of the triangle. The lengths of the three sides are a, b and c. The areas of the three squares are then  $a^2$ ,  $b^2$  and  $c^2$  respectively.
3. Now drag the vertices of the triangle ABC and observe the change. Collect five sets of data for the areas of the three squares. Write down the data in Table 11.1.

Set	Area of the smaller squares		Area of the greatest square
	$a^2$	$b^2$	$c^2$
1			
2			
3			
4			
5			

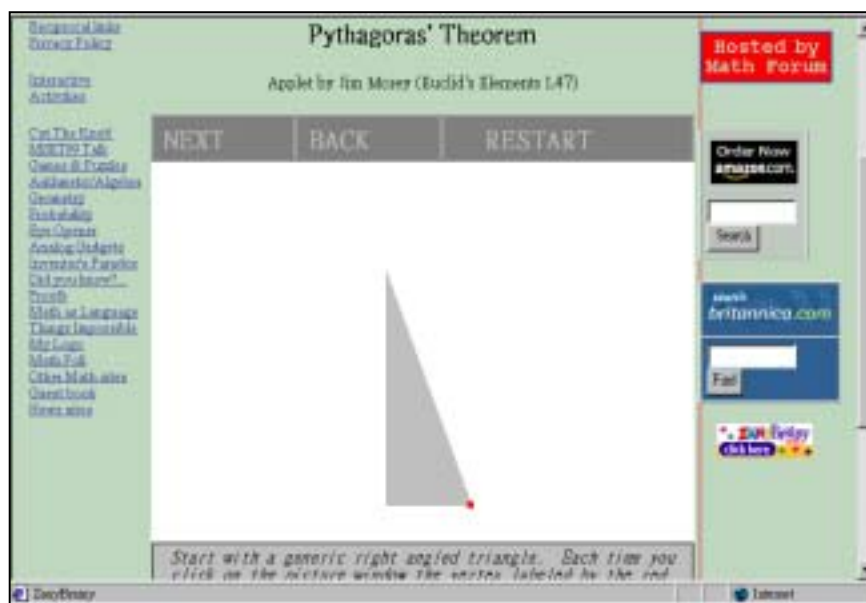
Table 11.1

4. Can you guess a relation between the areas of the three squares? Write down your conjecture below. You may discuss with your partner.

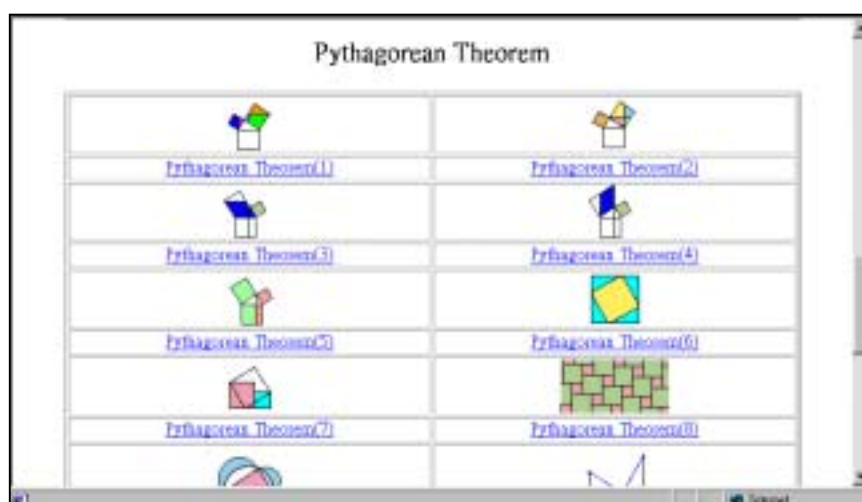
**Conjecture :**

## Part II : Proofs of Pythagoras' Theorem

5. Go to the web site <http://www.cut-the-knot.com/pythagoras/morey.html>.



6. Choose a right-angled triangle by following the instruction shown on the screen.
7. Press “NEXT” once to see the next step of the proof and notice the dialogue box below the figure until the completion of the proof.
8. Go to the web site <http://www.ies.co.jp/math/products/geo2/menu.html>. Try the interactive proof.



Internet

9. Go to the following web sites to find out other proofs of Pythagoras' Theorem.

- (a) <http://www.cut-the-knot.com/pythagoras/>
- (b) <http://www.math.ubc.ca/people/faculty/cass/Euclid/java/html/pythagoras.html>
- (c) <http://www.ies.co.jp/math/products/geo2/menu.html>
- (d) <http://www.geocities.com/capecanaveral/launchpad/3740/diagram.html>

Select two proofs other than those mentioned above that you feel easy to understand. Without downloading or copying the information directly from the Internet, write down the proofs in your own words below.

**Proof 1:**

**Proof 2:**

**Notes for Teachers :**

1. In this exemplar, students can come across and appreciate different ways of proving the same theorem.
2. In Part I of Worksheet 11.1 “Discovering Pythagoras’ Theorem”, students can make use of the interactive feature of the Java programme to produce different sets of data for the deduction of Pythagoras’ Theorem. Teachers should point out that there may be rounding errors in computing the three areas of squares.
3. The proof of Pythagoras’ Theorem can be done in two aspects. One is the formal proof by using paper and pencil while the other is the “visual proof”. For the abler students, they can write the formal proof by themselves. For the less able students, the visual proof is enough.
4. Students may not have time to come across all the proofs of Pythagoras’ Theorem, but they can still learn some more proofs from the presentations of other groups.
5. Description of the web sites:
  - a. <http://www.cut-the-knot.com/pythagoras/>  
contains many written proofs and links to other interactive proof
  - b. <http://www.math.ubc.ca/people/faculty/cass/Euclid/java/html/pythagoras.html>  
contains several interactive proofs, including the oldest one, using shearing, translation and similarity
  - c. <http://www.ies.co.jp/math/products/geo2/menu.html>  
contains 9 proofs of Pythagoras’ Theorem, some for demonstration while the others requiring students’ work
  - d. <http://www.geocities.com/capecanaveral/launchpad/3740/diagram.html>  
contains an interesting proof in the way of “moving-the-puzzle”
6. Suggested follow-up activity:

The teacher can ask students to do a project on Pythagoras’ Theorem. The project can focus on the authorship of the theorem, whether it is western or ancient China, and the comparison of different proofs.

7. Other useful web sites:
  - (a) <http://www.shodor.org/interactivate/activities/pyth/index.html>
  - (b) <http://www.utc.edu/~cpmawata/geom/geom7.htm>
  - (c) <http://home.netvigator.com/~wingkei9/javagsp/pythapf.html>
  - (d) <http://www.cmi.hku.hk/Teaching/Pytha/index.htm> (Chinese)
  - (e) <http://sm.hkcampus.net/~sm-lyk/Pyth.html> (Chinese)
  - (f) <http://sm.hkcampus.net/~sm-lyk/application.html> (Chinese)
  - (g) [http://www.edp.ust.hk/math/history/5/5\\_5/5\\_5\\_3.htm](http://www.edp.ust.hk/math/history/5/5_5/5_5_3.htm) (Chinese)
  
8. Some of the web sites mentioned in this exemplar may cease to exist or be relocated as time goes. The teacher should check and make sure that the web sites still exist before the activity.