Exemplar 4: Experimental Probability and Theoretical Probability

Dimension: Data Handling

Learning Unit: Simple Idea of Probability

Key Stage: 3

Materials Required: Spreadsheet software such as *Microsoft Excel* program and worksheets

Prerequisite Knowledge:
(1) Meaning of (theoretical) probability
(2) Tree diagram of outcomes in throwing three coins

Key Features:
The objective of this activity is to let students understand the meaning of experimental probability in contrast with theoretical probability. Through the traditional game "Scissors, Paper, Rock", students are asked to explore whether the game is fair.

In Part A, students have to play the game in a number of times as well as to simulate the case for a large number of trials with the aids of the software *EXCEL* to study the pattern of experimental probabilities.

In Part B, students will use the tree diagram they learned before to study the theoretical probability. Lastly, students are expected to make a contrast between experimental and theoretical probability.

All students have to participate in **both Part A and Part B of the Activity**. In Part B, teachers can use different worksheets to cater for different learning abilities of students. Uses of worksheets are summarized in the following table.

Part		Less able students	Average students	More able students
A	Worksheet 4.1	✓	✓	✓
B	Worksheet 4.2A	✓		
	Worksheet 4.2B		✓	
	Worksheet 4.2C			✓

Remark : ✓ represents the part(s) that can be participated by students when they start to learn the captioned topic.

In Worksheet 4.2A more guidance and structured questions are given to the less able students. Students would be guided to arrive at the conclusion that the game is fair.

In Worksheet 4.2B, less guidance or directed questions are set for average students. Students need to work on problems with additional constraints in playing the game and to determine whether the new game is fair or not.

In Worksheet 4.2C, more able students need to analyze the problem by themselves and make their own arguments. In addition, they have to work on more complicated questions with further added-on conditions in playing the game. Further, more able students need to modify the rules of the game to become a fair one.

Description of the Activity:

Part A: Playing the game "Scissors, Paper, Rock"

Introduction to the game:

This is an old game between two players. Each player makes his/her hand into either a "Scissors" (extend two fingers), "Paper" (palm flat down) or "Rock" (make a fist). The rule is: "Scissors" beats "Paper", "Paper" beats "Rock" and "Rock" beats "Scissors". If the two players form the same symbol, then the result is a tie.

1. Introduce the game "Scissors, Paper, Rock" to students and discuss with students the meaning of fairness of this game, that is, whether 2 players have equal chance of winning the game.

2. Ask students to guess the probability that A or B win the game from their past experience and, if possible, draw the consensus about the values (probably $\frac{1}{2}$ for both players).

3. Then ask students to play the game for a number of times so as to check the fairness of the game. The procedures are:
   (a) Divide students into pairs with one student as player A and the other as player B;
   (b) Ask students to play the game 10 times and record the results in Table 4.1 of Worksheet 4.1.

4. Ask some pairs of students to present their results and guide students to see that different pairs of students may have Player A (or B) getting quite different numbers of times in winning the game.

5. Guide students to observe the situation when the number of trials getting large by collecting all students' data. The teacher can consider to use *Excel* to calculate the percentages that A wins, B wins and getting a tie and to display the distribution of data to the whole class.

6. Raise, if necessary, the below questions for discussions:
   (a) What is the difference between the percentages of Players A and B in winning the game for individual groups (10 trials)? Why do the 2 players not have equal percentage as that we expected in the beginning of the game (refer to the discussions in Point 2)? Is it because Player A plays tricks in the game (for A has a higher percentage)? Why?

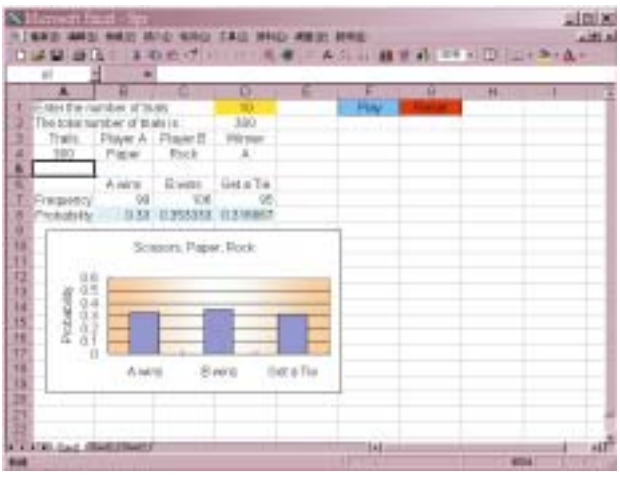
   (b) What is the difference between the percentages of Players A and B in winning the game for the total number of trials of the whole class?

   (c) Comparing the answers in (a) to that in (b), can you explain the phenomena?

   (d) Is there any relation between the percentages of 2 players in winning the game and the number of trials?

   (e) What do the percentages look like when we have a large number of trials? Is 200 trials (total number of trials of the whole class) sufficient large?

7. Distribute to students the spreadsheet program SPR.xls to simulate a large number of trials. Though students can simulate the game for several times with any number of trials, they preferably to try a large number of trials (at most 10 000 trials for each time; otherwise, it may take more than 3 minutes to run the simulation). They are asked to record the percentages that A wins, B wins or get a tie against the number of trials in Table 4.2 of Worksheet 4.1



8. Ask students to present their results and discuss the trend of the above three events when the number of trials increases.

9. Help students to draw conclusions that
   (a) the percentages that A (or B) wins in 10 trials, 200 trials or a large number of trials are called the experimental probability that A (or B) wins;
   (b) the experimental probability may not be the same for each time in playing the game;
   (c) however, the experimental probability tends to a certain value for a large number of trials (probably the same as the guess in Point 2);
   (d) the game seems to be fair to both players A and B if both of them are allowed to use "Scissors", "Paper" and "Rock" freely.

10. When there is a discrepancy between the guess in Point 2 and the empirical probability for a large number of trials, the teacher should lead students to do Part B of the Activity to find the theoretical probability. Students may follow Part B to justify their guesses.

Worksheet 4.1 Playing the game "Scissors, Paper, Rock"

Suppose you are player A. Your partner is player B. Play the game "Scissors, Paper, Rock" 10 times with your partner. Record the results in the table 4.1. Put a "✓" in appropriate place and count the total numbers and the corresponding % for each items.

Round	A wins	B wins	Getting a tie
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
Total			
%			

Table 4.1

Now open the *Excel* file SPR.xls. Run the game "Scissors, Paper, Rock" for a number of trials. Record your results in table 4.2.

Number of trials	A wins		B wins		Getting a tie	
	Number	%	Number	%	Number	%

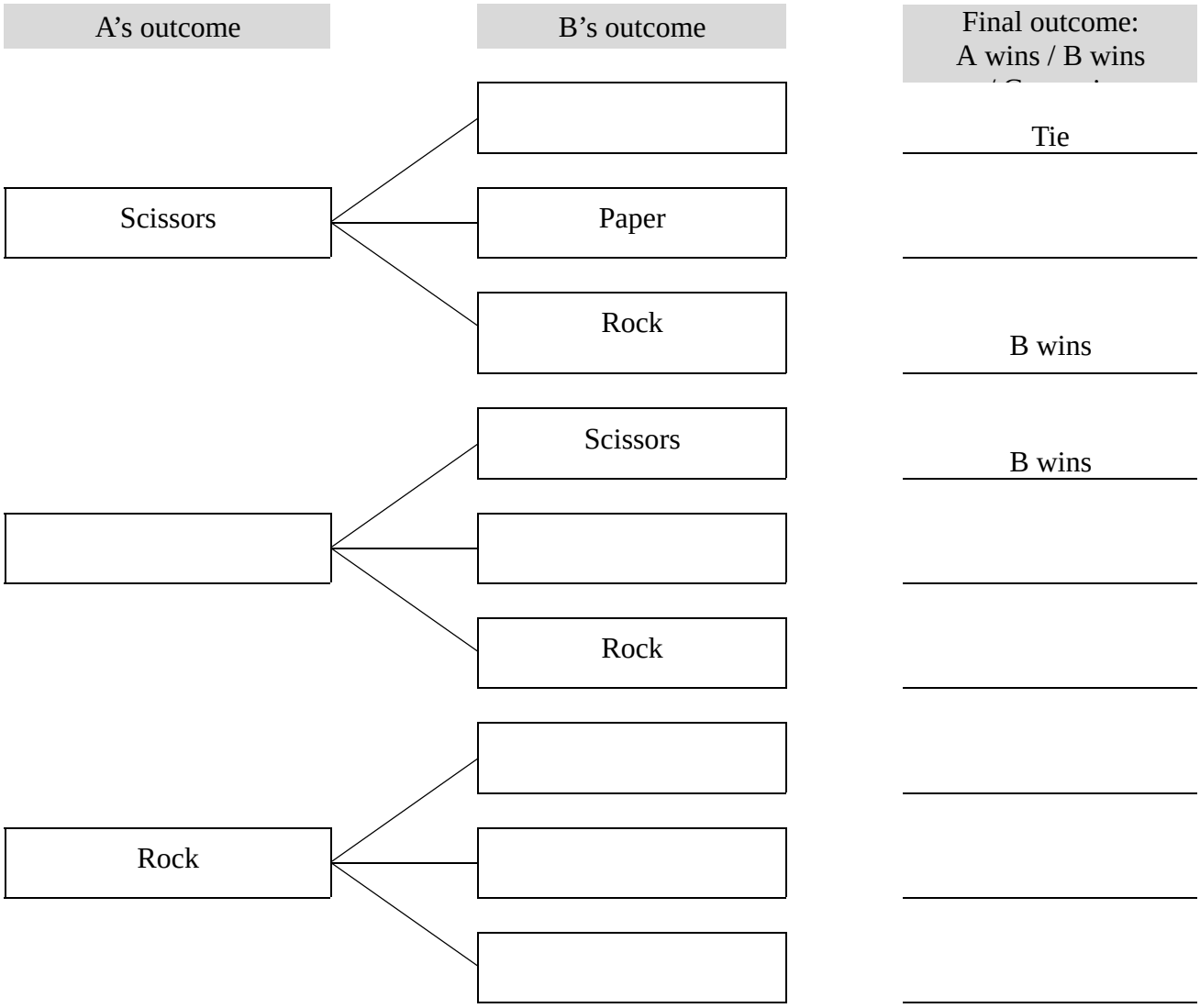
Table 4.2

Part B: Analysis of the game

1. Make a brief revision on the way to present outcomes in a tree diagram.
2. Divide students into groups of three or four students of similar abilities.
3. Distribute Worksheet 4.2A, 4.2B or 4.2C to groups of less able students, average and more able students respectively. They are asked to discuss the tree diagram of the game "Scissors, Paper and Rock" for 2 players A and B.
4. Ask students to complete or draw the tree diagram in Questions 1 and 2 of the Worksheets 4.2A and 4.2B or Question 1 in Worksheet 4.2C. The instructions in these worksheets are different to provide clues to students to solve the problem. For example, in Worksheet 4.2A, answers on the boxes along a branch may be provided as a hint to the less able students while an incomplete tree diagram with the omission of some of the branches may be given to more competent ones among the less able students.
5. For average and more able students, they may proceed to Question 3 in Worksheet 4.2B or Question 2 in Worksheet 4.2C respectively if they have finished previous questions of the Worksheets. The teacher may provide individual supports to these groups in answering these questions or they can ask the members of each group to help each other in studying the fairness of game in these cases.
6. Make a random selection of a student in one of the groups to come out and present the tree diagram of the group work. Another student (probably more able student) is invited to present his/her group's views on the fairness of the game in the case that "Scissors", "Paper" and "Rock" are freely used. He/she is expected to justify their guesses with reasons. The teacher should relate to the reasons provided by students with the answer in Question 2 of the Worksheet 4.2A or Worksheet 4.2B.
7. Introduce the term "theoretical probability" to students and compare the probability they guess in Point 2 of the Part A. Then the teacher discusses with students the differences between the empirical probability and the theoretical one.
8. Pass the answers of Question 3 in Worksheet 4.2B or Question 2 in Worksheet 4.2C to the corresponding groups for answer checking.
9. Points 1 to 8 are targeted at a class of mixed abilities. For the whole class consisting of more able students, for example, the teacher may consider to distribute only Worksheet 4.2C to them. Then the teacher should discuss in details with students all questions in Worksheet 4.2C especially the more difficult questions.

Worksheet 4.2A

1. Fill in the following tree diagram for the game “Scissors, Paper, Rock”.



2. Answer the following questions by considering the above tree diagram.
   (a) How many possible outcomes do the game have?
   (b) Count the number of wins for A.
   (c) From (b), find the probability that A will win in any round.
   (d) Count the number of wins for B.
   (e) From (b), find the probability B that will win in any round.
   (f) Count the number of getting a tie.
   (g) From (f), find the probability of getting a tie in any round.

Worksheet 4.2B

1. Write down a tree diagram below to analyze all positive outcomes of the game “Scissors, Paper, Rock”.

2. From the tree diagram in 1, find
   (a) the probability that A wins in the game;

   (b) the probability that B wins in the game;

   (c) the probability that getting a tie in the game;

3. How would the probabilities change if one player only used “ Paper” and “Rock”? Explain. Is the game fair in this case?

Worksheet 4.2C

1. Use a tree diagram to analyze the game "Scissors, Paper, Rock" to find out whether the game is fair. Write down your reasons below.

2. Answer the following questions.
   (a) How would the probabilities change if one player could only use "Paper" and "Rock"? Explain.

   (b) Now, there are three students playing the game "Scissors, Paper, Rock" together. The rules are as follows.
   (i) A wins if all 3 hands are the same.
   (ii) B wins if all 3 hands are different.
   (iii) C wins if exactly 2 hands are the same.
   Is this game fair? Give reasons.

   (c) Can you modify the rules of the game in part (b) in order to make the game to be fair? If possible, write down the modification.

Notes for Teachers:

Answers for Worksheet 4.2A:

1. Table for the game.

Case	A's outcome	B's outcome	Final outcome
1	Scissors	Scissors	Getting a tie
2	Scissors	Paper	A wins
3	Scissors	Rock	B wins
4	Paper	Scissors	B wins
5	Paper	Paper	Getting a tie
6	Paper	Rock	A wins
7	Rock	Scissors	A wins
8	Rock	Paper	B wins
9	Rock	Rock	Getting a tie

2. (a) 9 (b) 3 (c) $\frac{1}{3}$

(d) 3 (e) $\frac{1}{3}$ (f) 3

(g) $\frac{1}{3}$

Answers for Worksheet 4.2B:

1. Same as answers for question 1 in Worksheet 4.2A.

2. (a) $\frac{1}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{3}$.

3. Suppose that player B only used Paper and Rock.
   Table for the game (delete the case 1, 4 and 7 from the table in answers for question 1 in Worksheet 4.2A):

Case	A's outcome	B's outcome	Final outcome
1	Scissors	Paper	A wins
2	Scissors	Rock	B wins
3	Paper	Paper	Getting a tie
4	Paper	Rock	A wins
5	Rock	Paper	B wins
6	Rock	Rock	Getting a tie

   From the above table, we can see that the probabilities that A wins, B wins and Getting a tie are all equal to $\frac{1}{3}$. So the game is fair. Notice that the outcomes for A are random in regarding the new rule. Actually, we know that player B only used "Paper" and "Rock", it is impossible for A to use the "Rock" to beat the "Scissors" for player B. That means if A changes his/her strategy by using "Scissors" and "Paper" only, the probability that A wins that game will change to $\frac{1}{2}$ whereas that for B is $\frac{1}{4}$. (Delete the case 5 and 6 from the above table) .

Answers for Worksheet 4.2C:

1. Same as answers for questions 1 and 2, in Worksheet 4.2B.

2. (a) Same as answers for question 3 in Worksheet 4.2B.

(b) Table for all the 27 possible outcomes are listed as below:

Case	A's outcome	B's outcome	C's outcome	Final outcome
1	Scissors	Scissors	Scissors	A wins
2	Scissors	Scissors	Paper	C wins
3	Scissors	Scissors	Rock	C wins
4	Scissors	Paper	Scissors	C wins
5	Scissors	Paper	Paper	C wins
6	Scissors	Paper	Rock	B wins
7	Scissors	Rock	Scissors	C wins
8	Scissors	Rock	Paper	B wins
9	Scissors	Rock	Rock	C wins
10	Paper	Scissors	Scissors	C wins
11	Paper	Scissors	Paper	C wins
12	Paper	Scissors	Rock	B wins
13	Paper	Paper	Scissors	C wins
14	Paper	Paper	Paper	A wins
15	Paper	Paper	Rock	C wins
16	Paper	Rock	Scissors	B wins
17	Paper	Rock	Paper	C wins
18	Paper	Rock	Rock	C wins
19	Rock	Scissors	Scissors	C wins
20	Rock	Scissors	Paper	B wins
21	Rock	Scissors	Rock	C wins
22	Rock	Paper	Scissors	B wins
23	Rock	Paper	Paper	C wins
24	Rock	Paper	Rock	C wins
25	Rock	Rock	Scissors	C wins
26	Rock	Rock	Paper	C wins
27	Rock	Rock	Rock	A wins

From the above table, the probability that A wins = $\frac{3}{27} = \frac{1}{3}$.

The probability B wins = $\frac{6}{27} = \frac{2}{9}$. The probability that C wins = $\frac{18}{27} = \frac{2}{3}$.

(c) One of the modifications is that a player will win if his/her outcome beats all outcomes of other two players. Otherwise there is a tie.