Exemplar 8: Using Information Technology to Construct Tessellated Figures

Dimension: Measures, Shape and Space

Learning Unit: Angles Related with Lines and Rectilinear Figures

Key Stage: 3

Materials Required:
(1) Regular polygons made of translucent plastic sheets
(2) The equipment for students to access to the Internet
(3) Software such as *Paint* (Window NT 4.0 or above*)* and *Geometer's Sketchpad* (later referred as *Sketchpad*).
(4) Data Table for Regular Polygons (Table 1)
(5) Instructions for creating a tessellated figure through translation by *Paint* (refer to Annex I)
(6) Instructions for creating a tessellated figure through translation, reflection or rotation by *Sketchpad* (refer to Annexes II to IV)

Prerequisite knowledge:
(1) Simple idea of transformation including translation, reflection and rotation
(2) Sum of Interior angles of regular polygons

Key Features：

The objective of this activity is to let students understand the idea of tessellation in a plane and develop their understanding of the properties of lines, planes, angles and polygons. This exemplar consists of 3 parts. The teacher can organize learning activities summarising as follow to cater for their students' learning abilities.

Part		Less able students	Average students	More able students
A	Regular polygons	✓	✓	✓
	Other polygons	✓	✓	✓
B	Computer design		✓	✓
C	Project work			✓

Remark : ✓ represents the part(s) that can be participated by students when they start to learn the captioned topic.

Part A :

students are expected to:

- identify types of regular polygons that tessellate;
- explore which polygons can be used to tessellate.

Part B :

Students are asked to

- create one simple tessellated figure by using translation technique and computer **under the teacher's instructions.**

Part C :

Students are required to construct a tessellated figure as a group project. The figure should be constructed by using *Sketchpad* and involving more than one type of transformation. Students are expected to present their design in class. During group presentation, more able students of other groups can be invited to guess the types of transformation applied in constructing the figures.

Description of the Activity：

Part A

1. Demonstrate to students the following web sites for the exploration of various tessellated figures and introduce the idea of tessellation:

   http://forum.swarthmore.edu/alejandre/students.tess.html
   (Good tessellated figures made by students in other countries can be found in this web page)
   http://www18.big.or.jp/~mnaka/home.index.html
   (Interesting tessellated figures with animation effects can be found)

2. Group students in pairs. They are asked to tessellate a plane with sets of regular polygons made of translucent plastic sheets.

Data Table for Regular Polygons			
Regular Polygon	Number of Sides (n)	Each Interior Angle $\frac{180°(n-2)}{n}$	Tessellate a Plane? (Yes/No)
Equilateral Triangle			
Square			
Regular Pentagon			
Regular Hexagon			
Regular Heptagon			
Regular Octagon			

3. Ask students to explain their findings. Confirm students' findings that only 3 types of regular polygons, namely equilateral triangles, squares and regular hexagons, tessellate the plane.

4. Guide students to explain the findings with the focus on the interior angles of these figures. The following questions can be used to initiate their discussions:
   (a) What are the sizes of the interior angles of the regular polygons mentioned above?
   (b) Why are there only 3 regular polygons which can tessellate a plane?
   (c) Why is a regular pentagon not used to tessellate a plane?

5. Challenge students to investigate further what simple irregular polygons tessellate a plane. Students can cut different shapes of polygons by folding paper or other methods to test the conjectures mentioned below. Each group is expected to do one of the following experiments:
   (a) Try various types of triangles to tessellate a plane and discuss the conjecture "All triangles can tessellate a plane".
   (b) Try various types of quadrilaterals (some may not be convex). Discussion can be made on whether the conjecture "All quadrilaterals can tessellate a plane" is correct.
   (c) Try other types of polygons and challenge students whether the conjecture "All polygons can tessellate a plane" is correct.

6. Demonstrate students' work on the blackboard or on the transparencies with the help of an overhead projector. Guide students to confirm that the conjectures in (a) and (b) are correct.

Part B

1. Show a rectangle to students and demonstrate how to transform the shape by translation only and make the shape tessellate a plane by using the drawing software such as *Paint*.

2. Ask students to transform a square to tessellate a plane in a similar way. They may create the tessellation by using *Paint* or *Sketchpad* (refer Annexes I to IV). Figure 2 and Figure 3 are the illustration.

A tessellated figure created by *Paint* through translation

Figure 2

A tessellated figure created by *Sketchpad* through translation

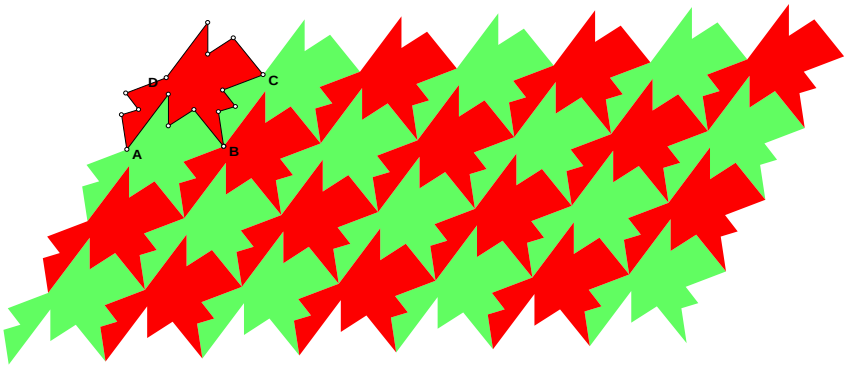


Figure 3

3. For more able students, they are advised to use *Sketchpad* to create the tessellated figures as it can help them apply the transformation skills and it is more flexible and powerful when compared with *Paint*.
4. For average students, they can use *Paint* to create the tessellated figure since it is quite easy to learn and it does not involve much transformation skill in creating tessellated figures

Part C

1. Demonstrate to students by transforming a polygon, including reflection and rotation, to form a tessellated figure.

   A tessellated figure created by *Sketchpad* through reflection and rotation

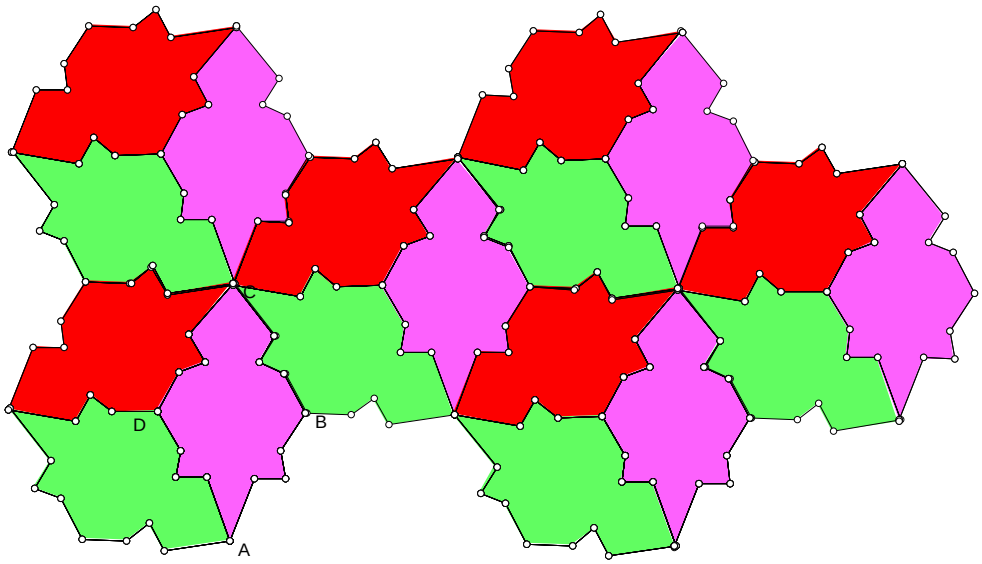


Figure 4

2. Ask students to use more than one type of transformation in designing the tessellated figure as a project. Students should decide which polygon (e.g. triangle, square, rectangle or hexagon) to start with and what transformations (translation, reflection and rotation) to be applied.

3. Give students the instructions for creating the figure through translation, reflection and rotation on a particular shape by *Sketchpad*.

4. Make a clear explanation on the requirements of the project and the deadline for submission. The following things are recommended to be included in the project:
   (a) Indicating which software is used and which web sites are quoted;
   (b) Showing the construction process step by step and the transformation techniques used in each step;
   (c) The print out of the tessellation.

5. Select and display some excellent work after the collection of the projects.

6. Invite students to present their ideas and techniques used to the class. Alternatively, teachers can invite more able students to guess the types of transformation applied in constructing the displayed products as a game activity.

Annex I

Guidelines for creating a tessellated figure through translation by *Paint*

Before getting started:

1. Starting the application and then select the **Image/Attributes** menu items. Select the "Inches" as "Units" by clicking on the appropriate button.

2. Set the attribute of the picture to inch, change the "Width" to 8.5 and the "Height" to 11 so that the empty image now conforms to a standard 8.5" x 11" format. This layout will provide adequate space for constructing and copying tessellation building blocks in the lower half of the page.

Step One

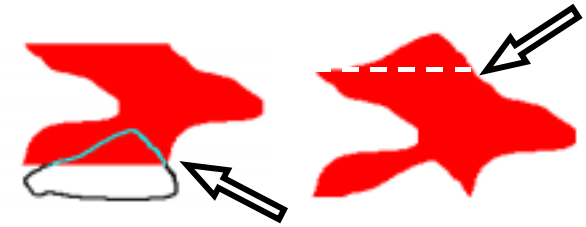
Select your favourite colour from the menu. Use the **Rectangle** Tool to draw a square (hold down the shift key when the "rectangle" is drawn). Since in *Paint*, the **Rectangle** Tool will only draw the outline of the square in the colour chosen, so one must also choose the **Fill With Colour** to make the square a solid as shown. It is recommended that you save your work frequently as you proceed through this activity. Save your creations as 256 colour bitmap (BMP) file.

Step Two

Using the **Free-Form** Select tool to cut the shape as you like. Once the loop is complete and the mouse button is released, the "cut" line will become dotted indicating the segment is "active". The dots will form a rectangular outline. Drag the active left side away from the initial square as shown. The active element will be transferred to the right side of the square as illustrated in the next step.

Step Three

Attach the "active" portion on the right side carefully.



Step Four

Repeat the process and start to cut at a bottom corner. Use this as a reference point when placing the bottom segment on the top of the shape.

Step Five

Use the **Pencil/Line** tools to decorate the figure. Do not add anything to the outer edges of the figure.

Step Six

Use the **Rectangular scissors** tool to draw a rectangle around the image. When a dotted frame surrounds the image, click on the **Edit/Copy** option. Next select **Edit/Paste** to position a second copy of the image on the screen. Drag this active image and position it to the right of the original as shown.

Step Seven

Change the colour of the image.

Step Eight

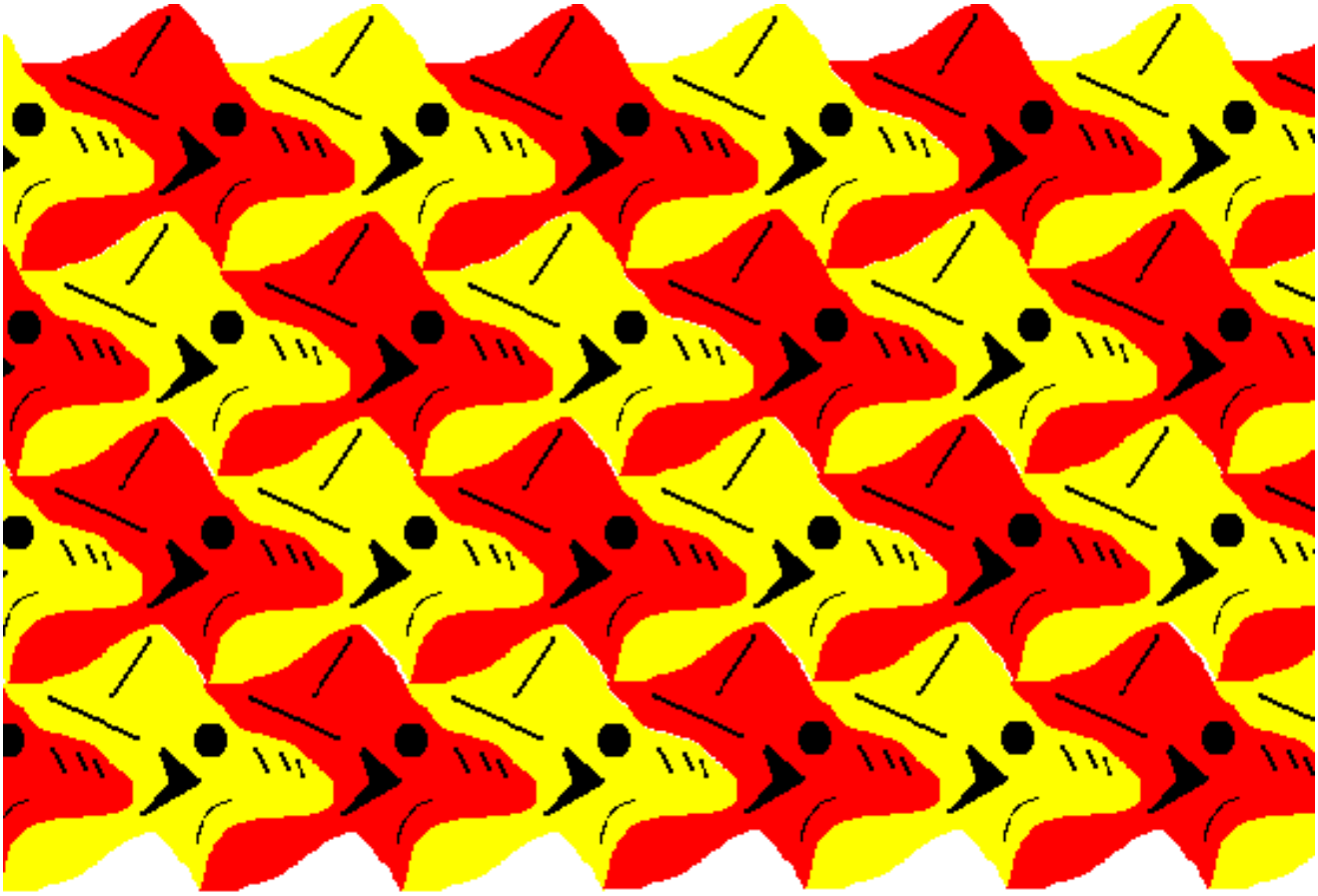
Merge two figures together. It becomes the basic building blocks for the tessellation.

Step Nine

By copying and pasting the image in turn, the first tessellation row is formed.

Step Ten

Once a row has been completed, you may copy an entire row and position them in a staggered format to create a series of rows down the page as illustrated below:



Annex II

Guidelines for creating a tessellated figure through translation by *Sketchpad*

The simplest way to create your own tessellation is by changing the opposite sides of a square, rhombus, hexagon, parallelogram, or rectangle.

(I) To Construct the tessellation pattern

1. Choose one of the polygons listed above. Draw the figure and colour it. Move the figure until it is oriented on the screen the way you want it.

2. Delete two opposite segments in the figure. Replace one of these segments with a wacky series of straight lines.

3. Select one of the vertices of your figure (an endpoint of the thing you just drew) and also the vertex that is directly opposite to it. Then select **Mark Vector** from the **Transform** menu.

4. Select all the segments and points that you just drew and nothing else. Make sure you DO NOT PICK THE VERTICES OF THE ORIGINAL FIGURE!

5. From the **Transform** menu, select **Translate** and click OK.

6. Repeat steps 2 - 5 until you have changed all the sides of your original figure.

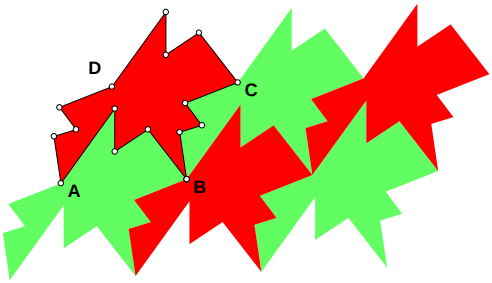
7. Choose all the vertices and then select **Polygon Interior** from the **Construct** menu. Change the colour of the interior of the figure from the **Display** menu.

(II) To tessellate the plane

1. Select any one of the vertices of your original figure and also the vertex that is directly opposite to it. Then select **Mark Vector** from the **Transform** menu.

2. Select the interior of the figure.

3. From the **Transform** menu, select **Translate** and click **OK**. You should have twice as many of those real shapes. Change the colour of the figure you just created.



4. Keep translating the figures until you have about 9 or 12 figures covering your sketch. You may need to mark a new vector occasionally. You can translate two or more at a time which will save you some time!

5. When you have finished, you need to put your name in a text box at the bottom of the sketch.

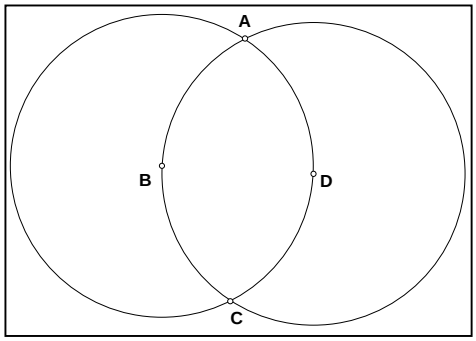
6. To print the figure, go to **Print Preview** and choose **Scale to fit on page**. Then send your tessellation to the printer.

Annex III

Guidelines for creating a tessellated figure through reflection by *Sketchpad*

To make a tessellation by reflection, you can start with a rhombus with 60° and 120° as the interior angles.

(I) To construct a rhombus



1. Place points B and D on the screen. Select B and then D. Make a circle by choosing **Circle By Center and Point** from the **Construct** menu.

2. Next select D and then B and make another circle by same process described in step 1.

3. Label the intersections of the two circles A and C.

(II) To create the tessellation pattern

1. The points A, B, C and D are the four vertices of your rhombus. You don't need to draw in the segments because you're going to hide them later anyway.

2. Connect points A and D with a series of segments (refer to the figure).

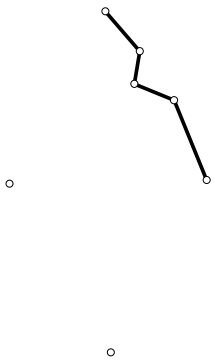
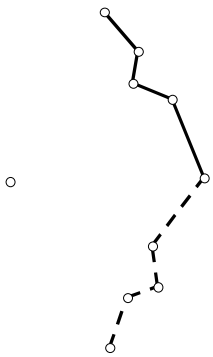
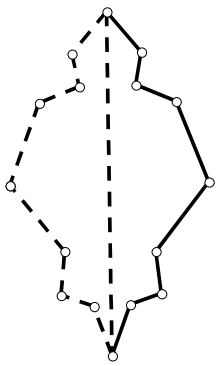
3. Select the vertex point D and mark it as the centre, either by double clicking or by using the **Mark Center "D"** from the **Transform** menu.

4. Select the new wacky shape you just created (but not the endpoints A and D) and choose **Rotate** from the **Transform** menu to rotate the segments around the point D by -120° (or 120° if that doesn't work). The shape should connect up with the next vertex point C (refer to the figure) .

5. Next, draw in the diagonal AC and mark it as a mirror, either by double-clicking or by choosing the **Mark Mirror** from the **Transform** menu.

6. Select the entire shape (but not the diagonal AC and not the vertex points A and D) and reflect it about AC (refer to the figure).

7. Hide diagonal AC and then drag the points around until none of the lines overlap.

		
Step 2 of (II)	Step 4 of (II)	Step 6 of (II)

(III) To tessellate the plane

1. Choose point D and mark it as the centre using the **Transform** menu.

2. Select the entire figure (draw a box around it using the **Arrow** tool) and use the **Transform** menu to rotate it by 120°.

3. While everything is still selected, rotate it again by 120°.

4. Choose another point (on the outer edge of the whole shape) that looks like the direct image of point D and mark it as the centre. (If you guess wrong, you can always go back using **Edit/Undo**)

5. Select everything in the picture and rotate it by 120°.

6. While everything is still selected, rotate it again by 120°.

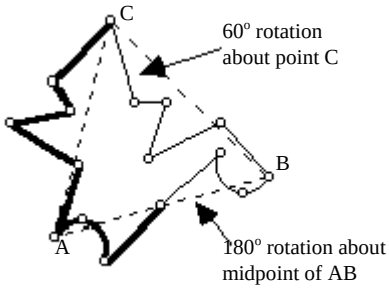
7. Continue steps 4 - 6 until you have enough shapes.

Annex IV

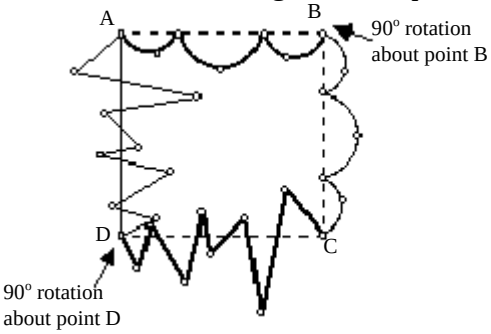
Guidelines for creating a tessellated figure through rotation by *Sketchpad*

To make a tessellation by rotation, you can start with an equilateral triangle, a square or a regular hexagon.

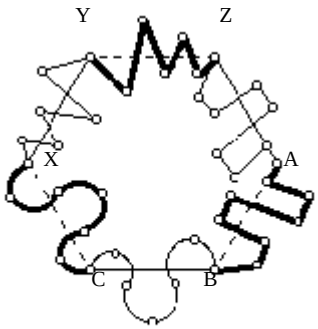
Follow this diagram for equilateral triangle.



Follow this diagram for square.



Follow this diagram for regular hexagon.



A image created using 120° rotations.
One about point Y, one about point A,
and one about point C

(I) To create the tessellation pattern

1 Connect the first two vertices, say A and B, with some series of segments and curves.

2 Choose point B and use the **Transform** menu to mark it as the centre.

3 Select all the points, curves and segments newly created (except for points A and B) and use the **Transform** menu to rotate them by a specified angle (60°, 90°, 120°, etc.) about point B.

4 **TRIANGLE**: Make a midpoint on one of the remaining sides of the triangle. Then connect the midpoint to one of the two adjacent vertices with some segments or curves. Select the midpoint as the centre and rotate newly created points, segments or curves about the midpoint by 180° (so it matches with the vertex on the other side).
SQUARE: Repeat steps 2 and 3 on the remaining two sides of the square.
HEXAGON: Repeat steps 2 and 3 twice, once on each remaining pair of adjacent sides.

(II) To tessellate the plane

1. Choose any vertex of the original figure and mark it as the centre using the **Transform** menu.

2. Select the entire figure (draw a box around it using the **Arrow** tool) and use the **Transform** menu to rotate it by 60°(triangle), 90° (square) or 120° (hexagon).

3. While everything is selected, rotate it again by the same amount. Repeat until that vertex is completely surrounded (a total of 6 images for triangles, 4 for squares, and 3 for hexagons).

4. Choose another point on the edge that is a vertex of the original shape and mark it as the centre.

5. Select the entire figure and rotate it by a suitable amount.

6. While everything is still selected, rotate it again.

7. Repeat steps 4 - 6 until you have enough shapes.

Notes for Teachers：

1. More instructions for creating a tessellation by *Sketchpad* can be found in the following web sites:
http://forum.swarthmore.edu/sum95/suzanne.tess.gsp.tutorial.html
http://www.mste.uiuc.edu/courses/ci336kt/garrison/skpdindx.html

2. An assessment form can be given to students so that they know the criteria for assessing their projects. The sample of the assessment form can be downloaded from the web site:
http://www.mste.uiuc.edu/courses/ci336kt/garrison/tessrub.doc

3. Students are not suggested to use *Paint* in their project work since there are some limitations in this software in the construction of tessellation through reflection and rotation. For example, the image can only flip horizontally by and vertically by and rotate by angles of 90°, 180° and 270° only. *Sketchpad* is more suitable to create a fine tessellated figure with more variations.

4. For more able students, the teacher can request them to write a written explanation about the construction process and the transformation techniques used in each step. Of course, even those students may find difficulty in writing the written explanation of the tessellation project. More guidelines should be given to them. The sample of the written explanation can be seen from the web site:
http://www.mste.uiuc.edu/courses/ci336kt/garrison.tesexpl.html

Reference Materials:

Web Sites:

1 http://library.thinkquest.org/16661/templates/index.html
2. http://library.thinkquest.org/16661/escher.html

Books:

1. Britton, Jill and Britton, Walter. (1992). *Teaching tessellating art: activities and transparency masters.* Palo Alto, California: Dale Seymour Publication.

2. Burn, Bob. (1987). *The Design of Tessellations.* Cambridge: Cambridge University Press.

3. Seymour, Dale and Britton, Jill. (1989). *Introduction to tessellations*. Palo Alto, California: Dale Seymour Publication.

4. Seymour, Dale. (1989). *Tessellation teaching masters.* Palo Alto, California: Dale Seymour Publication.