I. Introduction

Learners vary tremendously in their family background, parental expectation towards their performance, cognition, learning sequences, motivation towards learnings, their own perception on performance in mathematics and their role in the learning process. These factors constitute the cause and nature of learner differences[1]. They are variables for each learner and interact in a complex way. They affect teaching and learning activities and the quality of learning.

Upon the implementation of universal education in Hong Kong, a wider range of students gain access to secondary mathematics than has been in the past. As mathematics is a **subject for all primary and secondary students**, the issue of **learner differences** is especially marked. If our ultimate goals are to increase educational opportunities and maximize learning effectiveness for all our children, it must be accepted that educational policy makers, school administrators, teachers and educators need to be aware of the differences existing among learners and adopt appropriate measures to cater for these differences.

II. Strategies to Cater for Learner Differences

Although school teachers are always the ones who face the problems of learner differences in the classroom, it is not only their responsibilities to deal with these problems. 3 aspects in planning strategies to cater for learner differences are suggested below. They are: the central curriculum aspect, the school aspect and the classroom aspect.

A. Central Curriculum Aspect

In designing the central curriculum, **the needs of students at both ends** of the ability scale are **equally important**. Opportunities to learn should be maximized for all students. That is to say, attention should not be placed only on lower academic achievers. The needs of the more able students should also be catered for.

Because of the drawbacks of streaming at the early stage of schooling such as the labelling effect, it is resolved to have a single mathematics curriculum in the years of general education[2]. It targets to

---

[1] 鄧廣威、曾婉媚、陳瑞堅合編（1997）。《因材施教：教育上的特殊需要》。香港公開進修學院出版社。

[2] CDC Ad hoc Committee on Holistic Review of the Mathematics Curriculum (2000). *Report on Holistic Review of the Mathematics Curriculum*. Hong Kong:Printing Department, P.34 Para. 5.30

provide all students with the knowledge, skills and attitudes essential for a knowledgeable citizen of the modern age and the mathematical power, such as reasoning, for life-long learning. On the other hand, curriculum differentiation is adopted at the upper end of the secondary schooling to suit the different needs of students. Through curriculum differentiation, students would still be given opportunities for the access of mathematics based on different combinations of modules.

As a single subject for all students in general education, merely trimming down of learning contents is not a desirable way to address the needs of students in wide spectrum of ability even students at the lower end. Instead, flexibility in the curriculum organization is provided in the Syllabus for Secondary Schools: Mathematics (Secondary 1-5)(1999), (later referred as “the *Syllabus*”).

The **Foundation Part** of the *Syllabus* is identified. It contains the contents/skills which ALL students should strive to learn. Apart from the Foundation Part, teachers can judge for themselves the suitability and relevance of other topics in the **Whole Syllabus** for their own students. For more able students, teachers can adopt some **enrichment topics or organize enrichment activities** at their own discretion to extend these students’ horizon and exposure in mathematics

The Foundation Part is identified under the following principles:

(a) It is the essential part of the *Syllabus* stressing the basic concepts, knowledge, properties and simple applications in real-life situations;

(b) It contains different components that constitute a coherence curriculum.

Below is one example in the learning unit “Congruence and Similarity” in the Measures, Shape and Space Dimension of the *Syllabus*. The underlined learning objectives are considered as non-foundation of the *Syllabus* and the objectives with (*) are suggestions of enrichment topics. Teachers can have their discretion to select enrichment topics that suit their students’ needs and abilities. Enrichment topics closely related with the learning objectives in the *Syllabus* can be chosen, just like the first enrichment topic in the following exemplar. Alternatively, teachers can choose an independent topic not closely related to any topics in the *Syllabus* as enrichment, such as the second enrichment topic in the exemplar.

- Recognize the properties for congruent and similar triangles
- Extend the ideas of transformation and symmetry to explore the conditions for congruent and similar triangles
- Recognize the minimal conditions in fixing a triangle
- Identify whether 2 triangles are congruent/similar with simple reasons
- <u>Explore and justify the methods to construct angle bisectors, perpendicular bisectors and special angles by compasses and straight edges</u>
- <u>Appreciate the construction of lines and angles with minimal tools at hand</u>
- ** Discuss the possibility of trisecting an angle by compasses and straight edges
- **Explore some shapes in fractal geometry

For further details, teachers can refer to Chapter 4 of the *Syllabus*.

In order to provide teachers with further flexibility to organize the teaching sequences to meet individual teaching situations, the learning units and modules for each dimension are only subdivided into 2 **key stages** (KS), i.e. KS3 for S1 - S3, KS4 for S4 - S5. Teachers are free to design their school based mathematics curriculum for each year level with all learning areas suggested for each key stage in mind. An overview of learning modules and learning units in KS3 and KS4 can be found in Appendix 1.

Around 11% of **spare periods** are also reserved in designing the time allocation for the *Syllabus*. These spare periods create curriculum space for organizing consolidation activities or enrichment activities to suit the teaching approaches and the standard of students.

B. School Aspect

Although schools enroll students with a limited range of ability under the existing secondary school places allocation system, students in a school will, to a certain extent, still differ in mathematical abilities, needs and interest towards mathematics learning. The panel chairperson, in collaboration with other panelists, should make a careful diagnosis of the students' general strengths and weaknesses in mathematics as well as their needs. Based on this information, schools are expected, with close reference to the *Syllabus,* to draw up their school-based mathematics curriculum.

Strategies in catering for learner differences at the school levels include:

(a) Decide the aims and targets of the whole school mathematics curriculum and at each key stage.

(b) Adopt organizational arrangements such as providing additional lessons to certain students and ability grouping strategies like streaming, split class, withdrawal and cross-level subject setting. However, schools should be aware of the labelling effect and also the administrative workload in the arrangements. Nevertheless, enforcing students with different learning abilities, especially high ability students, to the same learning pace may lead to a decline in motivation and achievement. Schools should therefore adopt flexible-grouping strategies as far as possible. For example, allowing students to accelerate into other groups under the cross-level subject setting for a reasonable period of time such as a school term.

(c) Appropriately select the depth of treatment of the learning units that lie outside the Foundation Part of the *Syllabus* as the common core learning contents for all students. Flexibility should also be allowed for class teachers to select some other non-foundation topics in the *Syllabus* for more able students or consolidation activities for less able students.

(d) Arrange the learning units in a logical sequence for each year level taking the following into consideration:
   i. cognitive development and the mathematical abilities of students;
   ii. the affective elements of students;
   iii. the learning objectives of each learning unit;
   iv. the inter-relation of learning units (refer to the flowchart of learning units in Annex III of the *Syllabus*);
   v. the inter-relation of mathematical learning at different year levels; and
   vi. the total number of periods allocated for the mathematics subject in each school year.

(e) Choose an appropriate textbook and adapt or produce instructional materials. Schools may use different textbooks for different ability groups of students in the same year level, or use the same textbook but centrally produce different instructional materials to support students of similar ability in different classes. Both strategies have their strengths and weaknesses including labelling effects, workloads of teachers, copyright, etc. Schools have to consider their own background in making the decision.

(f) Design a wide variety of informal and non-formal learning activities such as statistical projects, weekly questions posted in the mathematics bulletin boards, mathematics books reading scheme, poster design using transformation of shapes, mathematics camp, Mathematics Olympiad, etc. Students with different inclinations and abilities may participate in different activities that suit their needs or interest. For example, schools could provide more challenging tasks for more able students to sustain their interest in the discipline. On the other hand, less able students can gain encouragement in participating in the said mathematical activities.

(g) Formulate the assessment policies and the method of recording and reporting to provide feedback for teaching and learning. Schools may empower teachers to design their own assessment methods to suit the needs of individual classes such as allowing certain percentages, say 5% to 10%, of the mathematics scores in students' report cards to teachers' own discretion. Class teachers may design their own test papers, project works, daily marks, etc., which account for 5% – 10% marks.

Exemplars of arrangement of learning units and activities to cater for different learning abilities of students at each year level can be found in **Appendixes 2 to 3**. Appendix 2 shows how the learning units can be organized at S1, having considered the depth of learning of each unit and the time allocation for the year and students' learning abilities. Appendix 3 shows how the periods can be allocated for activities in the Measures, Shape and Space Dimension at KS3 to cater for different ability of students. These exemplars are only for illustrative purpose and by no means exhaustive or implying a single solution for schools. Further, although these exemplars are for different learning abilities, schools could design strategies to cater for other types of learning differences such as different interests or backgrounds of students.

C. Classroom Aspect

No matter how the curriculum is written or how students are organized in schools, teacher is the key person in implementing the curriculum and to facilitate students' learning in the classroom. Therefore, it is important that the class teacher should be flexible enough to adjust his/her teaching plan to suit the needs of students.

Diagnosis of Students' Needs and Differences

First of all, teachers need to gather background information of students, including their interests, their strong and weak areas. The Hong Kong Attainment Tests and the proposed Basic Competency Assessment[3] may be used as a diagnostic instrument to help identify students' main areas of strengths and weaknesses. Self-designed tests can also be used for the same purpose. Teachers can also diagnose students' performance from students' profiles. Apart from written tests, teachers' own observation of students' performance in class and in written assignments is also a reliable basis for diagnosis.

Variation in Level of Difficulties and Contents Covered

Based on the above findings, teachers can plan relevant learning activities for each lesson. Teachers have to select, adapt or design materials to suit the range of abilities of their students. Too easy or too difficult tasks will not stimulate and sustain student's internal drive to learn. For less able students, tasks should be relatively simple and fundamental in nature. These activities can give students greater sense of satisfaction and hence greater confidence. For more able students, tasks assigned should be challenging enough to cultivate as well as to sustain their interest in mathematics learning.

Exemplars 1 to 8 in this booklet are developed to illustrate how teachers can design graded activities. In these exemplars, most students could work through the first part of the activities, Parts A and/or B, which are comparatively less challenging. Then some students could work through the other part of the activities (Part C). Exemplar 1 is a typical example in which Part C covers the non-foundation part of the *Syllabus*. For most of the activities, academically lower achievers often take more time in the first part of the activities so as to get a thorough understanding of the underlying concepts.

---

[3] Education Commission (2000). *Learning for Life. Learning through Life-Reform Proposals for the Education System in Hong Kong.* Hong Kong: Government Printing Department. See Appendix III.

They are not recommended to do the other parts of the activities although these activities may not be too difficult for them like Parts B and C of Exemplar 2 are on Fibonacci Sequence.

Variation in Questioning Techniques

On the other hand, teachers can, **through providing students with different clues** when asking questions, enable students to learn the same topic at the same year levels. In general, teachers can ask simple and straightforward questions to less able students and comparatively more challenging questions to more able ones. Even for the less able students, teachers can request them to modify their answers, to explain their strategies of solving the problems instead of giving the solutions right after they give wrong answers. The short dialogues between students and teachers can allow students and teachers to have immediate feedback from each other. Exemplar 3 is the example showing how teachers structure the questions and provide different verbal guidance to suit their students' abilities in the process of solving problems.

Variation in Clues provided in Tasks

Teachers can also provide students with the same task or exercise but with additional supports such as diagrams to aid comprehension and structuring long question for less able students. For the more able students, teachers ask open-ended questions and provide fewer hints in the process of solving problems (refer to Exemplar 4). The use of different worksheets in providing supports to different students can also be found in Exemplars 1 and 7.

Variation in Approaches in Introducing Concepts

Catering for learner differences can start from differences. These include the differences in cognitive development, the differences in experiences or interests. Hence, teachers can introduce mathematics concepts in different approaches. For the cognitive development aspect, teachers use more concrete examples to illustrate the concepts for students in concrete operational stage[4] as described by J. Piaget but can use symbolic language for students in the formal operational stage. Exemplars 5 and 6 illustrate how the same concepts can be conveyed in different approaches. Similarly, for the experiences or interests aspect, teachers can use more daily-life examples for some students but more abstract illustration for the others. For example, flowers from our environment are used to illustrate Fibonacci Sequence in Exemplar 2. Exemplar 5 also illustrates the use of daily-life situations to explain the rules of 4 operations in directed numbers.

Variation in Using Computer Packages

Different levels of exercises or activities are always included in the educational software packages. Appropriate use of information technology provides teachers with a way to cater for learner differences as it allows students with different abilities to learn at different paces. Further, the facilities to record students' performance in these software packages could also provide information for teachers to diagnose students' misconceptions or general weaknesses so that they can re-adjust the teaching pace or re-consider the teaching strategies. In addition, teachers can make use of the built-in functions in different software to design activities of different levels of difficulty. Exemplar 7 illustrates how teachers can use a single software package to engage students in tasks of different levels of difficulty. Exemplar 8 demonstrates how two different software packages can be used in designing the same activities with different levels of difficulty.

Variation in Peer Learning

Besides whole-class teaching, teachers can also consider different grouping strategies to cater for the needs of different students. However, it is important not to assume students sitting together in-groups can work collaboratively. Ingredients for successful collaborative learning include careful

[4] According to the theory of the Swiss psychologist Jean Piaget, human intellectual development progresses chronologically through 4 sequential stages: sensory-motor stage, pre-operational stage, concrete operational and formal operational stage. The ages at which people enter each higher order stage vary. Some secondary students may still in the concrete operational stage even when they are in junior secondary levels.

consideration on the formation of groups, the suitability of the tasks designed for the groups, the durability of the grouping and the ongoing assessment of the group dynamics. Further, it is very important to build up the collaborative instead of competitive atmosphere that is found undesirable in effective students' learning.

Teachers may consider grouping students with similar learning abilities, different learning abilities, or in different group size for collaborative learning. However, care must be taken to avoid labelling effect on grouping students of same abilities especially for a long duration. Research has found that the learning difficulties of students whose abilities are below average cannot be remedied from this type of grouping. Heterogeneous groups, on the contrary, are found leading both positive academic and remedial outcomes. When students of different abilities are grouped together, the high-ability students benefit from group interaction as much as the low- or average-ability students. Nevertheless, research suggests that for maximum communication among members, the ability range in the groups should not be too wide. In addition, groups of 3 to 4 students work well.

Importance in Arousing Learning Motivation

Among the factors in affecting learning performance, motivation is probably the most important one because a well-motivated learner is more determinative to achieve and it enables the learner to overcome a lot of difficulties. Motivation is not constant but may change according to the circumstance and disposition of the learner. Teachers must be aware of the possibilities of such changes and be flexible enough to adjust their strategies when necessary. The exemplars in this folder are only illustrations of adopting different strategies in catering for students with different learning abilities. It is **crucial** for teachers to modify the exemplars with particular attention paid to their own students' **interests** and to initiate their students' motivation.

Variation in Activities with Different Levels of Difficulty	Exemplars	Theme
Variation in Activities with Different Contents Covered	1	Investigation of Lines in Triangles
	2	Exploration of Fibonacci Sequence
Variation in Questioning Techniques	3	Constructing Data Set from a given Mean, Mode and Median
Variation in Clues provided in Worksheets	4	Exploration of the Meaning of Experimental and Theoretical Probabilities
Variation in Approaches in Introducing Concepts	5	Exploration of the 4 Basic Operations on Directed Numbers
	6	Exploration of the Formula for the Areas of a Circle
Variation in Using Computer Packages	7	Using Different Properties of Squares to Construct a Square with Information Technology
	8	Using Information Technology to Construct Tessellated Figures

An Overview of Learning Modules and Units

Number and Algebra Dimension

Key Stage 3 (S1 - S3)	Key Stage 4 (S4 - S5)
Number and Number Systems	
• Directed Numbers and the Number Line (12)	
• Numerical Estimation (5) • Approximation and Errors (7) • Rational and Irrational Numbers (6)	
Comparing Quantities	
• Using Percentages (17) • More about Percentages (7) • Rate and Ratio (8)	
Observing Patterns and Expressing Generality	
• Formulating Problems with Algebraic Language (14) • Manipulations of Simple Polynomials (10) • Laws of Integral Indices (10) • Factorization of Simple Polynomials (15)	• More about Polynomials (9) • Arithmetic and Geometric Sequences and Their Summation (10)
Algebraic Relations and Functions	
• Linear Equations in One Unknown (7) • Linear Equations in Two Unknowns (15) • Identities (8) • Formulas (14) • Linear Inequalities in One Unknown (7)	• Quadratic Equations in One Unknown (17) • More about Equations (15) • Variations (13) • Linear Inequalities in Two Unknowns (15) • Exponential and Logarithmic Functions (18) • Functions and Graphs (16)

Note: The number in the bracket denotes the estimated time ratio for the unit

Measures, Shape and Space Dimension

Key Stage 3 (S1 - S3)	Key Stage 4 (S4 - S5)
Measures in 2-Dimensional (2D) and 3-Dimensional (3D) Figures	
• Estimation in Measurement (6) • Simple Idea of Areas and Volumes (15) • More about Areas and Volumes (18)	
Learning Geometry through an Intuitive Approach	
• Introduction to Geometry (10) • Transformation and Symmetry (6) • Congruence and Similarity (14) • Angles Related with Lines and Rectilinear Figures (18) • More about 3-D Figures (8)	• Qualitative Treatment of Locus (6)
Learning Geometry through a Deductive Approach	
• Simple Introduction to Deductive Geometry (27) • Pythagoras' Theorem (8) • Quadrilaterals (15)	• Basic Properties of Circles (39)
Learning Geometry through an Analytic Approach	
• Introduction to Coordinates (9) • Coordinates Geometry of Straight Lines (12)	• Coordinate Treatment of Simple Locus Problems (14)
Trigonometry	
• Trigonometric Ratios and Using Trigonometry (26)	• More about Trigonometry (29)

Note: The number in the bracket denotes the estimated time ratio for the unit.

Data Handling Dimension

Key Stage 3 (S1 - S3)	Key Stage 4 (S4 - S5)
Organization and Presentation of Data	
• Introduction to Various Stages of Statistics (5) • Construction and Interpretation of Simple Diagrams and Graphs (24)	
Analysis and Interpretation of Data	
• Measures of Central Tendency (19)	• Measures of Dispersion (13)
Simple Statistical Surveys	
	• Uses and Abuses of Statistics (11)
Probability	
• Simple Idea of Probability (12)	• More about Probability (11)

4.2.4 Further Applications Module

Key Stage 3 (S1 - S3)	Key Stage 4 (S4 - S5)
	Further Applications (30)

Note: The number in the bracket denotes the estimated time ratio for the unit.

Exemplars of Arrangement of Learning Units at S1

For Schools with Majority Academically Lower Achievers

Year Level	Number & Algebra	Data Handling	Measures, Shape & Space	Sub-Total
S1	Revision of arithmetic * (+10) Directed numbers and the number line (12+3) Numerical estimation (5+2) Using percentages (17+3) Formulating problems with algebraic language (14+4) **48+22**	Introduction to various stages of statistics (5) Revision of statistical graphs learnt in primary schools & simple projects (+5) **5+5**	Introduction to Geometry (10+3) Transformation and symmetry (6+2) Congruence and similarity <u>excluding construction</u> (14–6+3)** Angles related with lines and rectilinear figures <u>excluding construction</u> (18–3+2) Estimation in Measurement (6) Simple Idea of Areas and Volumes (15+3) **69–9+13**	153@

For Schools with Majority High Ability Students

Year Level	Number & Algebra	Data Handling	Measures, Shape & Space	Sub-Total
S1	Directed numbers and the number line (12) Numerical estimation (5+2) Formulating problems with algebraic language (14+1) Laws of Integral Indices (10+3) Manipulation of Simple Polynomials (10) Linear Equations in One Unknown (7) **58+6**		Introduction to Geometry (10+3) Transformation and symmetry (6) Congruence and similarity (14+3) Angles related with lines and rectilinear figures (18+4) Estimation in Measurement (6) Simple Idea of Areas and Volumes (15+1) Introduction to Coordinates (9) **78+11**	153@

Note: *. The learning shaded are suggested learning contents to consolidate learning in primary school levels.
**. Numbers in brackets such as (14-6+3) are interpreted as follows:
14: number of periods as described in the *Syllabus*
-6: number of periods to be deducted because of not treating the topics in the non-foundation part.
+3: additional number of periods for enrichment or consolidation activities
@. The total number of periods allocated for each year level in KS3 is 160.

Exemplars of Activities and Arrangement of Learning Units in the Measures, Shape and Space Dimension in Each Year Level of KS3

For Class of Majority of Higher Ability Students

Level	Learning Units	Exemplars of Enrichment Activities
S1	Introduction to Geometry Transformation and Symmetry Estimation in Measurement Simple Idea of Areas and Volumes Congruence and Similarity Angles Related with Lines and Rectilinear figures Introduction to Coordinates	• Recognize the dual properties of some regular polyhedra • Explore and construct some semi-regular polyhedra, sometimes called Archimedian Solids as a project work • Use Information Technology to maximize the capacity of a container by cutting squares from the 4 corners of a sheet of A4 paper • Use Information Technology to explore and design some simple shapes in fractal geometry as a project work • Divide a line segment into *n* equal parts • Construct tessellated figures using Information Technology software such as *Sketchpad* as a project work
S2	Simple Introduction to Deductive Geometry Pythagoras' Theorem Quadrilaterals	• Write a simple report of Euclid and the contribution of his book "Elements" in the development of the study of geometry as a project work • Write a geometric proof of Pythagoras' Theorem from reading "Liu Hui's proof" • Study the Ancient attempts in using Pythagoras' Theorem and the property of similarity in finding the distance between the earth and the sun • Construct a square with *Sketchpad* by using the transformation techniques and the properties of quadrilaterals
S3	More about 3-D Figures More about Areas and Volumes Trigonometric Ratios and Using Trigonometry Coordinate Geometry of Straight Lines	• Investigate the reflectional and rotational symmetries in octahedron as a project work • Explore and deduce the formula for the surface area and volume of a sphere as a project work • Discuss and study the past attempts in finding the radius of the earth • Explore and derive the formula for external point of division • Write a report of the development of Cartesian System in the study of geometry and comparing it with Euclidean Geometry with an illustrative example such as the 2 approaches in proving the centroid dividing the median in the ratio of 2:1

Exemplars of Activities and Arrangement of Learning Units in the Measures, Shape and Space Dimension in Each Year Level of KS3

For Class of Majority Less Able Students

Level	Learning Units	Exemplar Activities
S1	Introduction to Geometry Transformation and Symmetry Congruence and Similarity excluding construction using straight edges and compasses Angles Related with Lines and Rectilinear Figures excluding construction Estimation in Measurement Simple Idea of Areas and Volumes	• Play a game, which is designed to name the geometric shapes learnt in primary schools • Use straws of different lengths to construct triangles for exploring the condition of congruence (S.S.S.) • Construct and design a tessellated figure by transforming shapes with the software *Paintbrush* • Explore the sum of interior angles and the sum of exterior angles of a triangle by moving the angles cut from paper triangles of different shapes • Use a pair of set squares to form specific angles such as 15°, 105°, 135°, etc. so as to have better sense of sizes of angles. • Construct shapes to approximate to the given dimensions and measures
S2	Introduction to Coordinates Simple Introduction to Deductive Geometry excluding lines and centres Pythagoras' Theorem Coordinate Geometry of Straight Line excluding internal point of division and rectilinear figures	• Play a game, which is designed to use coordinates in describing the location of students in the class • Make a presentation of the story of Euclid and his book "Elements" after reading teacher's suggested article • Prepare a model to illustrate the Pythagoras' Theorem • Read the story of the first crisis of mathematics and make an oral presentation
S3	Quadrilaterals excluding proofs and problems related with mid-point and intercept theorems More about 3-D Figures More about Areas and Volumes Trigonometric Ratios and Using Trigonometry	• Play a card game to match the properties of quadrilaterals with their corresponding quadrilaterals • Deduce the formula of the area of a circle by dissecting the circle into a certain number of sectors • Using Trigonometry to estimate the height of the hall or a nearby building, etc as a class activity