Appendix 1

Executive Summary of Research 1 – Comparative Studies of the Mathematics Curricula of Major Asian and Western Countries

This research was one of two related research studies commissioned by the Education Department on the mathematics curriculum. The other study was on the views of various sectors on the mathematics curriculum. The two studies constituted a coherent whole and their findings complemented each other. On the basis of the findings of both studies, recommendations were made in the other research report.

Background

In July 1997, an Ad hoc Committee on Holistic Review of Mathematics Curriculum was set up by the Curriculum Development Council (CDC) to conduct a holistic review of the mathematics curriculum in Hong Kong. Two supportive research studies were thus commissioned by the Education Department to provide inputs with sound academic principles and practical demand to the Committee in its work of reviewing the mathematics curriculum. It was anticipated that the analysis of the mathematics curricula of some major Asian and western countries and the strengths and weaknesses of Hong Kong students in comparison to students of other countries in this research would provide insight at the international level for local use.

Research questions and methodology

According to the objectives, the following research tasks and questions were set:

(a) To study the mathematics curricula of the major Asian and western countries with reference to their
   (i) curriculum documents including aims and objectives, content and depth of treatment of each domain, modes of courses;
   (ii) modes of assessment (both internal and external);
   (iii) implemented curriculum including scale of implementation, problems encountered and actual effort paid by their students in studying mathematics; and
   (iv) strengths and weaknesses of students in each country.

(b) To analyze the current mathematics curriculum of Hong Kong in comparison with and in contrast to overseas curricula in terms of
   (i) aims and objectives;

(ii) modes of courses;
(iii) content and depth of treatment of learning areas (both described in the document and in real practice);
(iv) modes of assessment (both internal and external); and
(v) strengths and weaknesses of local students in mathematics.

(c) To make recommendations on the
(i) aims and objectives of the future mathematics curriculum;
(ii) modes of courses;
(iii) modes of assessment (both internal and external); and
(iv) learning areas that need to pay more attention and learning areas that need to pay less attention.

In this research, document analysis and synthesis of previous studies were carried out in investigating the worldwide trends in mathematics education. Over 100 local and worldwide research papers and articles on the current state of mathematics education were reviewed and analyzed, providing background information on the strengths and weaknesses of Hong Kong students for the two related research studies. The aims and objectives, the contents and the modes of implementation of the mathematics curricula of the major Asian and western countries were compared with a reference of more than 30 curriculum documents from Hong Kong and seven other countries. Moreover, in the light of the relevant results of the Third International Mathematics and Science Study (TIMSS), mathematics teaching and learning, the achievement and the attitudes of Hong Kong students towards mathematics were investigated.

Curriculum trends and Hong Kong's mathematics education in the wider world

Compulsory education was an important and challenging issue confronting all countries. On the one hand, we should go for a "mathematics for all" curriculum, and on the other hand, mathematics teaching should cater for individual needs. Different countries had carried out different mathematics curriculum reforms to cater for the varied needs of individuals, and these reforms were designed according to their own traditions and cultures. Recent literature on the current state of mathematics education revealed that contemporary mathematics curriculum reforms focused on the approach rather than content. The enhancement of ability rather than skill as one of the goals of mathematics education, the impact of hi-tech, and individual differences were identified as the three major issues in universal mathematics education.

Aims and goals of mathematics education

There were two complementary categories of aims: those furthering the development of society and those furthering the development of the individual. The educational goals of mathematics curricula in 10 countries were summarized and classified into 3 categories, i.e. practical, disciplinary and cultural. In line with the worldwide trend, the task group of the Hong Kong CDC/Examination Authority Joint Working Party considered mathematics not merely as a technical tool, but also as an intellectual endeavour and a mode of thinking. It was believed that mathematics played a central role in human culture in a more general context.

On analyzing the mathematics curriculum standards of different countries, it was found that at the turn of the century, most of them were going for an expanding goal rather than aiming simply for acquisition of mathematical knowledge and skills. Application of mathematics in realistic problems, attitude of using mathematics in daily life problems, confidence in using mathematics, communicating with mathematics and appreciation of mathematics were receiving more emphasis. Quality education instead of coping-with-exam-education was stressed. Mode of thinking and high order abilities were promoted. In some countries, mathematical awareness and mathematisation were stressed. The mathematical calibre was mentioned and the continuation of the mathematics curriculum between levels was highlighted.

The relationship between "product" and "process" in mathematics curricula was widely discussed. Recent literature pointed out that there was no dichotomy between conceptual understanding and acquisition of basic skills. Both content and process were interactive elements of the curriculum and understanding was gained through techniques. Basic skills formed the foundation for the enhancement of high order abilities and their utilisation.

Generally speaking, the revised S1-S5 mathematics curriculum (1999) in Hong Kong was in line with the worldwide trends. Process abilities were very much emphasized in the West while basic skills and content were stressed in Asian countries. The Hong Kong mathematics curriculum attempted to strike a balance between process and content. However, further investigation for better linkage of the process and content was needed.

Impact of Information Technology

Information technology changed the mode of getting information and learning. The literature suggested that the role of information technology in the mathematics curriculum should be considered. In the era of information technology, skills were de-emphasized and more room was left for development of concepts.

Upon analyzing the mathematics curriculum of Hong Kong, it was found that the impact of information technology was also addressed though nothing was said on when and how the calculator and computer should be used to assist mathematics learning.

Individual differences

Ways in dealing with individual differences and mixed abilities among students was one of the major problems of universal education. In the era of universal education, a major concern of curriculum developers was how to maintain flexibility in a curriculum in order to cater for individual differences of students. Curriculum differentiation, the introduction of options of enrichment and remedial teaching were suggested in various educational systems in their mathematics curriculum reform.

Hong Kong was probably the place with the least flexibility and choice in its curriculum. At the curriculum structure level and the implemented curriculum level, a number of models were proposed in previous research studies.

Continuity of the Hong Kong mathematics curriculum

Continuity and consistency throughout the primary and secondary levels of the mathematics curricula were issues of concern in Hong Kong. In particular, the curriculum structure at the senior secondary level needed urgent attention. It was commented that the current mathematics curriculum document was too bulky. A curriculum framework from an epistemological perspective, which was conducive to learning, needed to be developed.

Content

The literature revealed that cultural factors played an important role in the mathematics curriculum of various countries. Asian countries and western countries treated the mathematics curriculum with different emphases and approaches. In Hong Kong, skills in solving mathematics problems were emphasized, whereas in London, care was given to individual difference. Asian countries and regions put a lot of emphasis on measurement, such as units, perimeter, area & volume, much more so than western countries. Coordinate Geometry was introduced at the junior secondary level in Hong Kong as a connection between Geometry and Algebra whilst the coordinate system was introduced with different approaches in the primary years in USA and UK. In East Asian countries, a "canonical" curriculum is stipulated by the governments which is followed closely in schools.

In Hong Kong, the introduction of topics into the curriculum was on average 2 years earlier than the international median. Basic skills and computations were some of the strong points

of mathematics education in the Chinese society. Hong Kong, Mainland China and Korea stressed computation ability in early grades. Some topics, such as Fraction and Decimal, were introduced, dealt with thoroughly and completed in primary school, whereas in other countries, these topics were continued to be dealt with in secondary school.

Nonetheless, the mathematics curriculum of Hong Kong was generally in line with the worldwide trends. Basic mathematical knowledge, skills as well as process abilities were stressed. Mathematics "senses" and applications were emphasized. Attitudes and affective factors for mathematics learning were taken into account.

Assessment

The dilemma between the goals of schooling for education and for selection, the shift from standards to standardization, the difference between quality of learning and checklist of learning outcome and curriculum control were widely discussed in the literature. It was commented that Asian countries should not simply import curricula from western countries since they had different cultural backgrounds. Mere change in curriculum document was not enough. More room for the professional development of teachers for better mathematics teaching should be made. De-emphasis of grades and downplay of examination culture were essential.

The literature pointed out that Confucian education, rote learning and examination driven-ness are not necessarily equivalent to each other. However, it was repeatedly found that Confucian education was often hampered by examinations. Examinations under the Confucian Heritage Culture (CHC) addressed low level cognitive goals and were highly competitive. Excessive pressure was exerted on teachers and students.

Students' perception of mathematics and learning styles

Success in mathematics was seen as a ticket of success for the future. It was reported in the literature that students in Hong Kong perceived mathematics as the solving of problems by simple methods, as a school subject having definite answers, as a subject of computations, as a set of rules, as a subject that requires thinking and as a useful subject with some parts not easily applicable to daily life. Students tended to identify mathematics by its terminology and subject content.

The literature revealed that CHC students had strong preference of surface approaches to learning. Practice was valued. This could serve as a firm foundation for the enhancement of high level abilities as long as practice and memorization go hand in hand with understanding, which could be deepened with practices with variations. CHC learners

possessed cultural potential for the deepening of understanding by means of repetitive learning. The excellent academic performance of Chinese learners was due to the synthesis of memorizing and understanding which was uncommon in the West.

Students' mathematics performance

Hong Kong had the highest population density among the 41 countries joining the TIMSS. It was among the countries with the largest class size and the least experienced mathematics teachers. Its education system was centralized, and spending for basic education was the lowest. Despite such adverse conditions, Hong Kong students did extremely well in the TIMSS mathematics tests. They outperformed students in most of the other countries, coming fourth behind Singapore, Japan and Korea. The superiority of the Hong Kong students over those in other countries became more marked when they proceeded from grade four to grade eight.

Although Hong Kong outperformed the western countries in mathematics in the TIMSS, it was less impressive when compared to East Asian countries. Furthermore, there were still a lot of simple and essential concepts and skills that many Hong Kong students failed to master. They performed significantly worse in the TIMSS Performance Assessment where students were required to conduct some hands-on activities.

Examiners' reports in Hong Kong public examinations revealed that the major problems for Hong Kong students were their inadequacies in tackling problems and in their mental processes, rather than a lack of skills or basic knowledge. Students were generally weak in tackling non-routine exploratory questions. Many candidates were unable to make use of the information to solve problems. Logical reasoning and handling problems involving a variety of topics were weak.

Students' attitudes

Mathematics achievement was found to be closely related to affective variables, especially attitude towards mathematics and self-concept, which in turn were related to self and parental expectation. The literature showed a high correlation between students' interest in mathematics and their achievements. Based on the analysis of the relevant result of the TIMSS, it was found that Hong Kong students in general had high regard for mathematics as it was often perceived as a major subject. Hong Kong students in both grades 4 and 8 thought that it was important to do well in mathematics. This was further enhanced by the high value placed on constant practice and painstaking effort on the part of students which were significant features of the Chinese culture. However, Hong Kong students did not show exceptional interest in mathematics despite their high regard for it.

Primary school students' attitudes towards mathematics, as perceived by the teachers, were much better than the attitudes of those in secondary schools. In general, they did not believe in luck having anything to do with high achievement while teachers in Hong Kong tended not to believe in natural talent. Similar to other TIMSS countries, students in Hong Kong agreed that it took a lot of hard work to do well in mathematics. On the contrary, they considered sports or having fun less important compared to students in other parts of the world. Influenced by the achievement orientation of the Chinese culture, students made effort in their studies. However, not too much time in the school curriculum had been devoted to mathematics in Hong Kong compared to other parts of the world. This might be due to the relative emphasis on the second language in the Hong Kong curriculum.

The high achievement of Hong Kong students might have been achieved at the expense of other aspects of the development of the students. One possible trade-off of Hong Kong students' superior achievement in mathematics was their relatively low confidence in mathematics. Students did not display the corresponding level of positive attitudes and confidence towards mathematics. It was found that students' self-concept of learning mathematics dropped as they advanced through the grade levels.

One's confidence and self image were something that was reinforced by one's learned values, and if students were constantly taught to rate themselves low, they might internalize the idea and might result in really low confidence. Rigid and conformed modes of learning added extra hardship. The examination orientation reinforced the status quo. A feeling of alienation led to taking the mathematics class as boring, and paying great effort without arriving at desired outcomes generated frustrations. Attitude towards mathematics began to deteriorate and a lack of confidence finally led to giving up the subject.

The greatest concern was that Hong Kong students did not have much confidence in doing mathematics. In general, girls had a lower perception of their ability than boys. In sharp contrast to their high achievement, many students in Hong Kong did not think that they did well in mathematics. This may be a result of the Chinese culture on the virtue of humility or modesty, but it may also well be a result of the competitive examination system and a culture of lack of encouragement on the part of the teachers.

Classroom practice

Research revealed that Hong Kong students spent over one-third of their homework time on mathematics homework though the time spent had low correlation with their mathematics achievement. Classroom lecturing was the most common mode of instruction. In contrast

to western countries, which were more flexible in their use of textbooks, East Asian countries put a lot of emphasis on the textbooks. Most of the teachers in Hong Kong took textbooks as reference in preparing for their lessons, and they hesitated to perform curriculum tailoring. However, the textbooks in Hong Kong focused much of their attention on the performance expectations of "knowing" and "using routine procedures". The emphasis on examination further reinforced learning by rote.

The teacher

The teacher as "the authority in the classroom" phenomenon, which was often regarded as a hampering effect, and a mixture of authoritarianism and student-centredness in the CHC classroom was identified. It was evident that students hoped for a lively approach in teaching. Students valued a teacher who showed concern for them. The teacher was the key figure in student learning, especially for young students. The teacher's personality was a decisive factor of the students' liking of the subject. A good teacher was the most crucial factor leading to a good mathematics classroom as well as successful curriculum implementation. Teachers' ownership of the curriculum, sharing of educational goals and professionalism should be the prerequisites for curriculum change. The teacher's professionalism and development were at the heart of any curriculum reform and implementation. The TIMSS results revealed that the reform of mathematics curriculum in Japan took place in classroom teaching rather than mere revision of curriculum documents. More time was spent on applying mathematics concepts and thinking in the Japanese classrooms. The Japanese experience of "lesson study group" was worth exploring.

Conclusion

In a pluralistic and highly technological society, mathematics should be taught as a subject which possesses several different goals that reflect the diverse roles mathematics plays in the society. Students constructing their mathematical knowledge with an expanding goal is the world trend in mathematics curriculum reform. The essence of a curriculum lay in the approach rather than the arrangement of contents.

The current bulky mathematics curriculum needs to be re-examined to leave room for deep understanding and enhancement of high order thinking abilities but we should safeguard going for a watered-down curriculum. Adjustments made on the choice of teaching/learning activities and depth of treatment in individual topics are preferred rather than the simple addition and deletion of topics. In order to leave room for cultivating the motivation of student learning and the development of high order abilities, the scope and depth of the current curriculum content have to be re-considered.

Mathematics curriculum should aim for expanding goals. Mathematical knowledge, concepts, problem solving skills as well as abilities to discover and invent should be encompassed. Affective domains such as attitude and confidence, high order thinking abilities should also be taken care of. High order thinking should be addressed and cultural aspects of mathematics should not be overlooked in the future mathematics curriculum.

Due consideration should be given to both mathematical skills (content) and the process of learning (ability), as well as to the use of technology in mathematics teaching. Careful consideration should be given to examination orientations. The development of assessment of high order thinking is a world issue, but we should, at the same time, safeguard against having the curriculum driven by examinations. Of paramount importance was the promotion of student's interest in and understanding of the subject. As individual differences and mixed abilities are identified as major issues of concern for mathematics education in the next century, flexibility of the curriculum is necessary to cater for the above. All these new ideas stated above cannot be accomplished without the professionalism of mathematics teachers. Hence, a favourable curriculum should offer a far-reaching vision and also room for teachers to exercise their teaching at their best.