Chapter 5

The Anticipated Mathematics Curriculum

Learning Dimensions in the Mathematics Curriculum

5.1 The idea of using dimensions (with other names, such as strands, stems, domains, attainment targets) is widely used in the design of various types of mathematics curricula in the world (see Appendix 5 for a few examples). Designing the mathematics curriculum in a framework of learning dimensions enables the learning objectives and students' progress to be structured and represented systematically from primary through secondary levels.

5.2 The mathematics curriculum should be designed so that content knowledge can serve as a means to develop an awareness of mathematics among students and they can gradually be able to conceptualize mathematically phenomena and problems met in daily life or in other disciplines. Furthermore, they can frame and formulate these problems in the mathematical language, to solve them with mathematical tools and then to make sense out of it in the mathematical way. In order to have this attained, the following points should be considered:

(a) The mathematics curriculum should be designed in such a way that mathematics learning progresses from concrete to abstract.

(b) The content in the mathematics curriculum should be arranged in such a way that students could get adequate prior experience with concrete objects before formal treatment of mathematical concepts. Abstract concepts should also be backed up by an abundance of mathematical and non-mathematical (daily-life) examples.

(c) Pre-mature exposure should be avoided at all costs.

(d) Prior experience of informal formulation of problems, such as sketches, diagrams, tabling of data, is necessary for formal problem solving.

(e) The arrangement of content should progress along various dimensions, for instance, starting from the manipulation of numbers and gradually transiting to the manipulation of symbols; from handling of shapes to describing location and then to spatial abilities in 2- and 3-dimensions; from the measurement with objects that can be directly measured to those which can only be measured indirectly and finally to those abstract measures such as measures of likelihood and central tendency; from handling information to problem solving and interpretations and inferences.

5.3 The pace of different dimensions is arranged so that pre-requisite knowledge is sufficiently tackled. Since the focus of learning and the particularity of learning different domains are emphasized, we think that the integration of various dimensions is not a viable arrangement among the dimensions. In fact, connection could be dealt with in teaching and integration should be left to application problems.

Recommendations

5.4 The mathematics curriculum should be designed according to a set of content-based learning dimensions so that learning objectives and students' progress can be structured and represented systematically from primary through secondary levels. However, it is not necessary to extend the use of dimensions in designing sixth form mathematics curriculum.

5.5 The mathematics curriculum should be designed in such a way that mathematics learning progresses from concrete to abstract. The content in the mathematics curriculum should be arranged to let students get adequate prior experience with concrete objects before the formal treatment of mathematical concepts. Abstract concepts should also be backed up by an abundance of mathematical and non-mathematical (daily-life) examples.

5.6 Since HOTs can only be developed through the learning of various mathematical knowledge in the content areas, they should be incorporated into the content-based learning dimensions to form a reference grid in designing the future mathematics curriculum (see Appendix 6).

5.7 Teachers should be aware of those mathematical learning experiences, like social mathematics, mathematics appreciation, history of mathematics, which are not easily defined within the framework.

Cross-level Interface in the Mathematics Curriculum

5.8 In the existing situation of HK, mathematics is a core subject from P.1 to S.5. At the senior secondary and sixth form levels, an Additional Mathematics syllabus and 4 mathematics syllabuses are respectively available for students who are interested in mathematics, while at the pre-primary level, preliminary exposure with mathematics is also experienced. Since kindergarten learning is usually not assumed, the primary mathematics is expected to start almost from scratch. However, formal treatments in teaching mathematics in some kindergartens are often found. This naturally results in a continuity problem for students upon admission to P.1.

5.9 At present, the school mathematics curriculum is more content-oriented. The mathematics curriculum of HK is a bit tight and difficult, and is sometimes criticized being not related to real life. Teaching, particularly at the senior primary, senior secondary and sixth form levels, when facing high-stake assessments, is examination driven. Rote learning and drilling exercises are the most commonly used means in class teaching.

5.10 The mathematics curriculum at S.1 repeats some of the materials at P.5 and P.6 to allow some time for students to adapt to the change in the medium of instruction. The Additional Mathematics Syllabus is relatively long when compared with that of the Secondary Mathematics and is not in proportion with the allocated teaching time. Some overlapping with the Secondary Mathematics Syllabus is also found.

5.11 The poor continuation between the senior secondary and the sixth form curricula is also a concern to most senior level secondary teachers. Typical examples include Additional Mathematics, AS-level Mathematics and Statistics and A-level Pure Mathematics. The A-level Pure Mathematics curriculum is abstract, difficult and lengthy. According to the regulation of the public examination, Additional Mathematics is not a prerequisite of the A-level Pure Mathematics. However, it would be extremely difficult to study Pure Mathematics without Additional Mathematics. Consequently, students strive for studying Additional Mathematics though they do not have the ability. The target students of AS-level Mathematics and Statistics is not clear. Those who have taken Additional Mathematics find the AS-level Mathematics and Statistics too easy while Arts students find it too difficult. Moreover, the mathematics and statistics parts are not fully integrated.

5.12 The mathematics curriculum should be reviewed as a whole with particular attention paid to the smoothening of the interface at various levels of schooling to ensure coherence and continuity of the curriculum. Mathematics learning at the kindergarten level should be activity-based and as intuitive and simple as possible, and arouse students' interest towards learning and the curiosity they already have. In primary and secondary schooling, mathematics learning should go from concrete to abstract and mathematical skills should not be separated from its content. The practice of skills should go alongside with understanding. An abundance of concrete real-life and mathematical examples are necessary for the acquisition of more abstract notions. Primary mathematics should be considered as a foundation stage for the development of students' mathematics knowledge and skills while the secondary mathematics curriculum should serve as a continuation and extension of the primary mathematics curriculum. More emphasis should be put on training students' logical reasoning and critical thinking, and on developing students' mathematical concepts rather than on developing the computational skills only.

5.13 The mathematics education at the primary and secondary levels (including teaching content, teaching strategies and assessment) should be considered as an entity. Great care and measures should be taken to ensure cross-level linkage within different dimensions at the primary and secondary levels. Connection across dimensions should also be considered. The structure of the mathematics curriculum at the upper end of the secondary schooling should be reorganized to ensure continuation and suit the different future mathematics needs of students.

Recommendations

5.14 Different levels should be considered as a continuous learning process. Objectives across various levels should be coherent and allow the development of mathematical concepts from concrete to abstract. The learning objectives and students' progress should be structured and systematically represented within and across levels.

5.15 The acquisition of process abilities should be allied with the learning of mathematical content. Topics in the mathematics curriculum should be arranged into progressive learning dimensions so that students' cognitive development in mathematics can be enhanced in a more coordinated way and the relevance of students ensured. For instance, the teaching of abstract mathematical ideas should be supported by students' concrete experiences at earlier stages as far as possible. Students need time to play around with concrete objects before proceeding to more abstract notions at the senior level.

5.16 The mathematics curriculum should be developed as a whole, but it should be duly adjusted across different levels of learning to cater for the different abilities of students. It is up to teachers' discretion to re-visit some topics when needed. However, it should be noted that though the spiral approach has its strong points, teaching too many topics in a single year and making learning fragmented should be avoided. In addition, some measures like organizing bridging programmes should be taken to ensure that students of different abilities can follow. In the curriculum development process, a flow chart indicating the inter-relation of topics of different mathematics curricula at different levels is recommended to ensure continuity.

5.17 Teaching strategies should be progressively changed through different levels of schooling, say from concrete to abstract, so as to cope with students' development. Teaching at P.1 should selectively adopt a thematic approach. Similarly, investigational work is encouraged at all the primary levels and should be continued in secondary schools.

5.18 Continuity should not be restricted to the curriculum document level. Teachers at different levels should be well informed on what is going on at the others. Measures should be taken to reinforce interflow between primary and secondary mathematics teachers.

Mathematics for Early Childhood Education

5.19 The aim of the early childhood education is to provide children with a balanced development in the moral, intellectual, physical, social and emotional aspects. The curriculum should include elements assisting young children to adapt themselves to social life and to understand the relationships between the individuals and society. Children should be educated in a natural and pleasant environment through various theme-related learning activities involving life experiences, for example, role-play and simulation games.

5.20 As children have already acquired some mathematical knowledge informally through their daily life experiences before entering schools, the introduction of mathematics in early childhood education is to help children enrich and supplement their informal mathematical experience in a meaningful way. It provides children with an opportunity to expose themselves to the experience of learning mathematics, which would be beneficial to them in learning mathematics at the primary level in later years. It also gives children an opportunity to develop their ability to apply mathematical concepts and skills in daily life situations. However, the formation of early mathematical concepts is only part of the general learning activities in the pre-primary curriculum. It should be conducted in integration with other learning activities. We stress that undue emphases on subject teaching should be avoided. A formal approach in mathematics teaching and learning is undesirable at this stage.

5.21 Mathematics in the early childhood education should introduce to children the basic knowledge in mathematics according to their ages, experiences and interests through various kinds of mathematical activities. The objectives of introducing early mathematical concepts are to

(a) stimulate children's interest and motivation in learning mathematics and cultivate in children a positive attitude through learning mathematics;

(b) help children develop their concentration and memory, observational and analytical abilities, and problem solving skills; build up the preliminary concepts of number, shape, space, data handling, time, computation and measurement; and develop their ability to communicate mathematical ideas and experiences.

Recommendations

5.22 Since children at the pre-primary level are only expected to acquire basic skills and concepts in mathematics, they should not be formally assessed and no prerequisite academic knowledge should be expected of them on their admission to P.1. Heuristic methods of teaching should be adopted to help children foster an interest in learning mathematics and develop an inquiry mind. Thematic approach with integrated activities of learning should be used for the purpose of flexible curriculum integration.

5.23 Mathematical concepts should be introduced according to the developmental stages of children, their interests and needs, by using simple language, and through learning activities which involve manipulation of objects and relate to children's experience. A mathematically enriched environment is essential for the children to work in. Teachers should see that there is an ample amount of relevant materials available for the children to manipulate and experiment with.

5.24 Children should be given the opportunities to explore, discover and develop mathematical ideas. Importance should be given to the process of learning mathematics rather than the outcomes of the activities. Flexibility should be allowed in planning teaching programmes and designing learning activities to cater for the learner differences, and incidental teaching should be adopted.

5.25 Children should be encouraged to discuss and describe their daily encounters in an appropriate mathematical vocabulary. Mathematical jargons like statistics, sum and difference, etc. which are beyond children's understanding should not be used. Over-teaching, premature exposure to mathematical knowledge and abstract concepts should be avoided as they may hamper children's development of learning mathematics.

Mathematics Curriculum for Post-Basic Education

5.26 As students move along the grade levels, they use mathematics at different extent. This is particularly pronounced at the upper end of the secondary schooling. Students may go for further studies in different disciplines such as science, technology, commerce and social sciences and some may join the work force in different professions.

5.27 In the existing situation, students in S.4 and S.5, who are more able in mathematics, may choose to take the subject Additional Mathematics which aims to provide better

foundations for studying Sixth Form mathematics and strengthen students' critical mathematical thinking. Since some of the content areas in this subject has overlapped with those of Secondary Mathematics, AS-level Mathematics & Statistics and A-level Pure Mathematics, previous knowledge of Additional Mathematics would definitely be an advantage to students studying the AS-level and A-level mathematics subjects although it is not explicitly required. This has not only brought some difficulties to teachers teaching the subjects concerned but has also generated an unnecessary inclination among students in taking Additional Mathematics even they do not have the competence.

5.28 Moreover, the similarity between the two Applied Mathematics subjects, the lengthiness of A-level Pure Mathematics and the unclear target students of AS-level Mathematics & Statistics have begotten grievances from the school teachers.

5.29 We hold that the mathematics curricula at the senior secondary and the sixth form levels have to be re-structured. The re-structuring should achieve the following objectives:

(a) To provide appropriate learning experiences for students of different abilities and orientation.
(b) To rest upon strong theoretical foundation including the pedagogical consideration and the consideration of the knowledge structure of mathematics itself.
(c) To ensure continuation and to avoid overlapping between the senior secondary and sixth form levels in mathematics.
(d) To broaden the possibility of studying mathematics at the sixth form level for all students.
(e) To prepare students for future education along different tracks.

5.30 We do not agree to have curriculum differentiation in the years of general education. Instead, mathematics should be a subject studied by all students. This subject should enable students to develop a mathematics literacy for the information age, the basic competence in mathematics and the foundation mathematics needed for a knowledgeable citizen. The identification of foundation and non-foundation parts is good enough to cater for learner differences. More details are provided in Chapter 6.

5.31 To cater for the needs of different students at the upper end of the secondary schooling, a number of subjects, which may come from different combinations of modules and papers, should be designed for students who aim to further their studies in the mathematics related fields. Streaming students at an early age should be avoided and opportunities for taking mathematics at the senior levels should be allowed for all students.

A Proposed Model of the Mathematics Curriculum

5.32 Based on the assumption that there is no change in the academic structure of the secondary schooling (that is, there are 5 secondary levels and 2 sixth form levels in the educational system), we recommend to have only one mathematics curriculum at the senior secondary levels with the subject Additional Mathematics deleted for the purpose of achieving equality of access and for reducing the labeling effect. This "general mathematics curriculum" should consist of foundation and non-foundation parts together with enrichment topics like the revised secondary mathematics curriculum to cater for the learner differences.

5.33 At the sixth form level, we propose to offer 6 modules: 4 on mathematics (i.e. Math1-Math4) and 2 on statistics (i.e. Stat1-Stat2). The depths and breadths of these modules are described in the following table:

Module	Similar in depth and breadth to
Math1	The mathematics component of current AS-level Mathematics & Statistics
Math2	Current CE Additional Mathematics minus Math1
Math3 + Math4	Current A-level Pure Mathematics minus current CE Additional Mathematics
Stat1	The statistics component of current AS-level Mathematics & Statistics
Stat2	The statistics component of current A-level Applied Mathematics minus Stat1

Note: Some mathematics topics in the current A-level Applied Mathematics such as differential equations and numerical methods will be absorbed into the 4 modules on mathematics.

5.34 The recommended combinations of these 6 modules are:

Subject	Modules
AS-level Mathematics	Math1+Math2
AS-level Statistics	Stat1+Stat2
AS-level Mathematics & Statistics	Math1+Stat1
A-level Mathematics & Statistics	Math1+Math2+Stat1+Stat2
A-level Mathematics	Math1+Math2+Math3+Math4

Note: Students may choose any one of these five subjects or the combination "A-level Mathematics + AS-level Statistics".

A diagrammatic representation of the model is shown in Appendix 7.

5.35 No module should be counted more than once in the public examination concerned. For example, students are not allowed to choose AS-level Statistics and AS-level Mathematics & Statistics at one sitting as the module Stat1 is common to both papers. On the other hand, students taking the modules Math1, Math2, Stat1 and Stat2 should pre-determine that they are counted as one A-level Mathematics & Statistics subject or as two AS-level subjects (i.e. AS-level Mathematics and AS-level Statistics). To avoid confusion, schools should consider offering subjects instead of modules. We believe that the above model can be easily adjusted even if there are changes in the academic structure of the educational system.