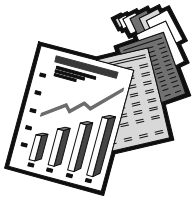

Exemplar 11 :

A Look at the Average Wage

Objective : To distinguish the differences in the interpretations of the mean, median and mode of a set of data.

Learning Unit : Measures of Central Tendency

Key Stage : 3

Materials Required : Scenario for the case study and calculators

Prerequisite Knowledge : Calculation of mean, median and mode

Description of the Activity :

1. The teacher distributes and explains the scenario for the case study to students.

2. The teacher distributes the worksheet to students.

3. Students work through the first two questions individually.

4. Students are then divided into groups of 3. Each group investigates the case given in Question 3.

5. Among the three students in each group, one student represents the president of the company, another one represents the Union Head and the last one represents the general clerk.

6. Based on the viewpoints and job natures of these characters, students are asked to discuss which measure of central tendency they support and give reasons.

7. The groups then report to the class.

8. Creative and reasonable views from students should be valued in order to provoke their thinking from various perspectives.

9. The teacher summarizes the relative advantages and disadvantages of mean, mode and median. The teacher points out that using these averages to analyze a set of data depends very much on the nature of the task and the presumptions laid by the analyst.

10. The teacher may suggest students to bring some articles from newspapers and other sources and evaluate the measures of central tendency in the following lesson.

Scenario :

The Head of the Employees' Union in a company was negotiating with Mr. Chan, the president of the company on the issue of salary adjustment. The Union Head said, “Our workers need more money to cope with the rising cost of living. No one in our Union earns more than $17 500 a month.”

Mr. Chan replied, “It's true that costs are going up. It's the same for the company—we have to pay higher costs for raw materials, so we get a lower profit. Besides, the average salary in our company is $19 000. I don't see how we can afford a wage increase at this time.”

The Union conducted an urgent meeting in that evening. A general clerk said, “We make only $7 500 per month. The workmen representing the largest sector of the population in this company make $12 000. We want our pay to be increased to that level at least.”

The Union Head decided to take a detailed look on the salary information. He went to the payroll department and got the information as shown in the table below.

Type of job	Number employed	Salary($)	Union member
President	1	150 000	No
Vice-president	2	90 000	No
Director	3	60 000	No
Branch Manager	3	45 000	No
Supervisor	3	30 000	No
Foreman	6	17 500	Yes
Payroll Clerk	3	14 000	Yes
Secretary	6	12 500	Yes
Workman	30	12 000	Yes
Sales Clerk	15	8 000	Yes
General Clerk	6	7 500	Yes
Total	78	1 482 000	—
$\text{Mean} = \$\frac{1\,482\,000}{78} = \$19\,000$			

The Union Head thought, “Mr. Chan is right, but the mean salary is pulled up by those executives' high salaries. Therefore, it does not give a really good picture of the salaries of the majority of staff. The general clerk's request is reasonable. Each of the thirty workmen makes $12 000. That's the most common salary level — the mode. However, there are twenty-one union members who make less than $12 000.”

Finally, the Union Head said to himself, “I wonder what the middle salary should be.” He thought of the employees as being lined up in order of salary, from low to high. The middle salary (it is called the median) is midway between employee 39 and employee 40.

Worksheet : A Look at the Average Wage

Referring to the scenario, answer the following questions:

1. Who will favour the use of (a) the mean, (b) the mode and (c) the median in this case study?

2. If the salaries of the 21 clerks with the lowest salaries were raised to $12 000, find
   (a) the new mean,

   (b) the new mode,

   (c) the new median.

3. If all salaries were raised by the same amount,

   (a) which measure(s) of the central tendency would remain the same?

   _____

   _____

   _____

   _____

   _____

   _____

   (b) which measure(s) of the central tendency would change? Why?

   _____

   _____

   _____

   _____

   _____

   _____

   _____

   (c) If only one or two clerks' salaries were increased,

      (i) which measure(s) of the central tendency would definitely change?

      _____

      _____

      _____

      _____

(ii) which of the other measure(s) of the central tendency would most likely remain unchanged?

_____

_____

_____

_____

_____

_____

(iii) how likely would the median be affected?

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4. If the president of the company raised his own salary only so that the mean salary was increased by $1 000? What is his new salary?

_____

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_____

5. The company lay off one foreman and two workmen. Predict whether the mean salary will increase, decrease or remain the same. Give reasons.

_____

_____

_____

Notes for Teachers :

1. Suggested answers for Question 1:
   For the discussion of the standpoint the students support, there is no single correct answer. Management would naturally favor the mean; the Union Head, the median; and the lower-paid union members, the mode.

2. Suggested answers for Question 2:
   If the salaries of the twenty-one clerks with the lowest salaries were all raised to $12 000, then
   (a) the new mean ≈ $20 115.38;
   (b) the new mode = $12 000;
   (c) the new median = $12 000.

3. Suggested answers for Question 3:
   (a) The mode and the median would remain the same.
   (b) The mean would change since its calculation included all values.
   (c) (i) The mean would definitely change since its calculation involved all the data.
   (ii) The mode would most likely remain the same because it was the most frequently occurred salary. Its value would not probably be affected by changing only one or two salaries.
   (iii) If the median was embedded in the middle of several salaries that were the same, it would not change. If the median was close to a different level of salary, it was likely to change.

4. For Question 4, the president's salary will be increased by $78 000 if the mean salary is increased by $1 000. His new salary is $228 000.

5. For Question 5, the mean salary will increase since the three employees being sacked have salaries below the original mean.
   The new mean salary = $(1 482 000 – 2×12 000 – 17 500) ÷ 75 = $19 206.6

6. Students may use spreadsheets to explore the changes of the mean, the median and the mode salary in the activity.

7.

	Mean	Median	Mode
Consider all the data	✓	✗	✗
Easy to compute	✓	✓	✓
Influenced by extreme values	✓	✗	✗
Uniqueness	✓	✓	Can be more than one

8. For newspaper articles and other sources of data summaries from the real world, the median is often reported instead of the mean, especially when extremely high or low values are included in the data sets. By using a variety of contexts and examples, teachers can help students understand when to use the mean, median and mode to represent the central tendency of data sets.