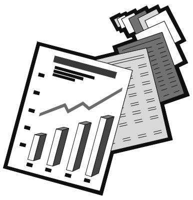


Exemplar 15

A Game of Dice – "6" to Win

Objective : To consolidate the relation between the empirical and theoretical probabilities

Learning Unit : Simple Idea of Probability

Key Stage : 3

Materials Required : Dice

Prerequisite Knowledge : (1) Meanings of theoretical probability and empirical probability
(2) Listing outcomes of events in tables

Description of the Activity :

1. The teacher divides students in pairs and assigns one student as ***Player 1*** and the other student as ***Player 2***. Each group is assigned a group number.

2. One die and two worksheets are distributed to each group. Players play the game according to the following rules:

> ***Player 1*** *and* ***Player 2*** *throw the die in turn.*
> *If one player gets a "6" but the other player does not, then the player who gets a "6" wins and the player who does not get a "6" loses.*
> *If both players get the same number, the game is drawn.*
> *If both players obtain numbers other than "6", the game is drawn.*

3. Each group plays the game 20 times and the results are recorded in Table 1 of the Worksheet.

4. After playing the game for 20 times, students find the empirical probability of winning from the results they obtained. Students compare these probabilities among themselves.

5. Each player copies his/her own set of results from Table 1 to Table 2. ***Player 1*** (or ***Player 2***) collects four more sets of results from ***Player 1*** (or ***Player 2***) of other groups. He/She goes on to calculate the empirical probability of winning, drawing and losing the game by gradually increasing the number of trials from 20 to 100.

6. Students are asked to list all the possible outcomes in a table. They then find the theoretical probability of winning, drawing and losing from the table.

7. Students are asked to draw a conclusion on the relation between the theoretical probability and the empirical probability.

8. The teacher guides students to discuss whether there are diffferences in the probabilities of winning, drawing and losing between ***Player 1*** and ***Player 2***.

Worksheet

Group ____ Player ____

Each group gets one die.
Both players play the game according to the following rules:

Player 1 *and* ***Player 2*** *throw the die in turn.*
If one player gets a "6" but the other player does not, then the player who gets a "6" wins and the player who does not get a "6" loses.
If both players get the same number, the game is drawn.
If both players obtain numbers other than "6", the game is drawn.

1. Record the results of 20 throws in the table below.

Trial number	Put ✓in the appropriate box		
	Win	Draw	Lose
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
Total no. of ✓			

Table 1

2. Using the results in Table 1, answer the following questions.

   (a) Total number of "Win" = __________.
   Total number of "Draw" = __________.
   Total number of "Lose" = __________.

   (b) (i) What is the probability of winning a game?

   ____

   (ii) What is the probability of having a draw?

   ____

   (iii) What is the probability of losing a game?

   ____

3. ***Player 1*** (or ***Player 2***) collects four more sets of results from ***Player 1*** (or ***Player 2***) of other groups and the data are summarized in the table below.

Number of trials	Number of “win”	Probability	Number of “draw”	Probability	Number of “lose”	Probability
20						
40						
60						
80						
100						

Table 2

4. For each of the probabilities of “win”, “draw” and “lose”, what do you observe as the number of trials increases?

____

____

5. List all the possible outcomes of the game in the following table. Express the outcomes in the form (x , y), x being the outcome of ***Player 1*** and y the outcome of ***Player 2***.

	Player 2					
Player 1	(1 , 1)	(2 , 1)				
	(1 , 2)					

Table 3

6. Using the results in Table 3, answer the following questions.

   (i) What is the probability of winning a game?

   (ii) What is the probability of making a draw?

   (iii) What is the probability of losing a game?

7. Comparing the results in Questions 3 and 6, what can you conclude about the relation between the probabilities obtained from the trials (the empirical probability) and the probabilities obtained from lisiting the possible outcomes (the theoretical probability).

Notes For Teachers :

1. After explaining the rules of the game, the teacher can ask student what they think about the chance of winning before starting the game. After the activity, students can then check whether their guesses were right.

2. In case students have difficulties in answering Question 4 because the data in Question 3 are not significant, then the teacher can collect more data from the other groups to make the sample size larger. Students may find it easier to discover the results from this larger set of data.

3. Answers to Question 5:
   The outcomes are listed in the form (*x* , *y*) in the table below, *x* being the outcome of ***Player 1*** and *y* the outcome of ***Player 2.***

	Player 2					
Player 1	*(1 , 1)*	*(2 , 1)*	*(3 , 1)*	*(4 , 1)*	*(5 , 1)*	*(6 , 1)*
	(1 , 2)	*(2 , 2)*	*(3 , 2)*	*(4 , 2)*	*(5 , 2)*	*(6 , 2)*
	(1 , 3)	*(2 , 3)*	*(3 , 3)*	*(4 , 3)*	*(5 , 3)*	*(6 , 3)*
	(1 , 4)	*(2 , 4)*	*(3 , 4)*	*(4 , 4)*	*(5 , 4)*	*(6 , 4)*
	(1 , 5)	*(2 , 5)*	*(3 , 5)*	*(4 , 5)*	*(5 , 5)*	*(6 , 5)*
	(1 , 6)	*(2 , 6)*	*(3 , 6)*	*(4 , 6)*	*(5 , 6)*	*(6 , 6)*

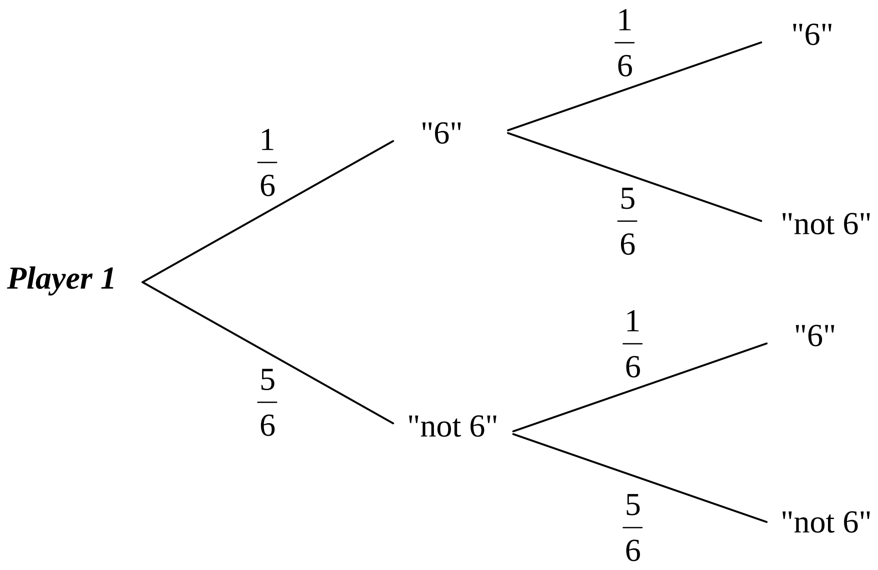
4. Answers to Question 6:

   (i) $\frac{5}{36}$

   (ii) $\frac{26}{36}$

   (iii) $\frac{5}{36}$

5. Alternatively, the teacher may guide students to find the answers to Question 6 by a tree diagram.
   The tree diagram for ***Player 1*** is as follows:



For ***Player 1***, P(winning the game) $= \frac{1}{6} \times \frac{5}{6}$

$$= \frac{5}{36}$$

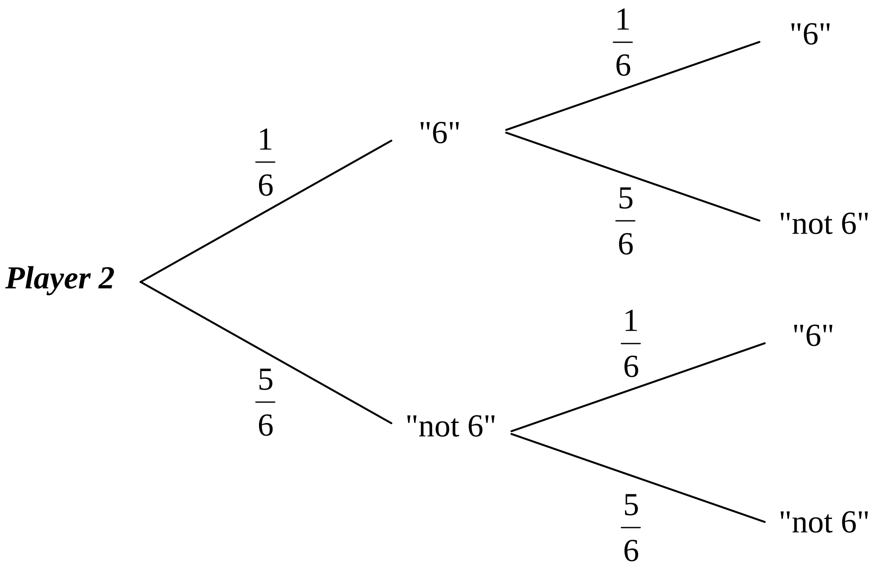
P(drawing the game) $= \frac{1}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6}$

$$= \frac{26}{36}$$

P(losing the game) $= \frac{5}{6} \times \frac{1}{6}$

$$= \frac{5}{36}$$

6. Similarly, the tree diagram for ***Player 2*** is as follows.



For ***Player 2***, P(winning the game) $= \frac{1}{6} \times \frac{5}{6}$

$= \frac{5}{36}$

P(drawing the game) $= \frac{1}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6}$

$= \frac{26}{36}$

P(losing the game) $= \frac{5}{6} \times \frac{1}{6}$

$= \frac{5}{36}$

7. For less able students, the teacher can ask students to fill the cells in Table 3 in different colours to represent “win”, “draw” and “lose” so that they may find it easier to work out the probabilities.

8. For more able students, the teacher can ask them to modify the rules of the game and try the new game with their classmates.

9. The teacher may guide students to compare the different theoretical probabilities of the two players on winning, drawing and losing the game to see if there is any relation between them. Students should see that the theoretical probabilities of ***Player 1*** and ***Player 2*** on winning, drawing and losing are identical.