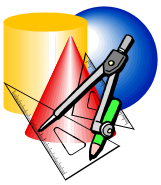

Exemplar 16:

Symmetries in 3-D Shapes

Objective: To recognize and appreciate symmetries in cubes and regular tetrahedra

Key Stage: 3

Learning Unit: More about 3-D Figures

Materials Required: (1) Sets of Platonic solids made from drinking straw or transparent acrylic
(2) Computer for demonstration and files in 3D_Sym

Prerequisite Knowledge: Basic concept on reflectional and rotational symmetry on a plane

Description of the Activity:

1. The teacher asks students to recall the meaning of reflectional and rotational symmetry of 2-D figures.

2. Then teacher shows a cube and invites students to interpret the idea of reflectional symmetry and rotational symmetry of the cube. The teacher can then show the symmetries by reflecting/rotating the cubes or use the provided files Ref_cube.exe and Rot_cube.exe in folder 3D_Sym to demonstrate the movement. The teacher then summarizes the meaning of reflectional symmetry and rotational symmetry of 3-D solids (See Fig. 1 and Fig. 2).

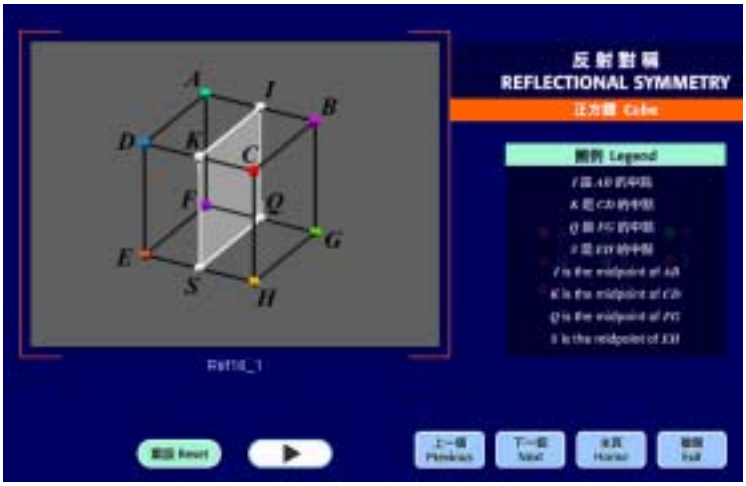

Fig. 1

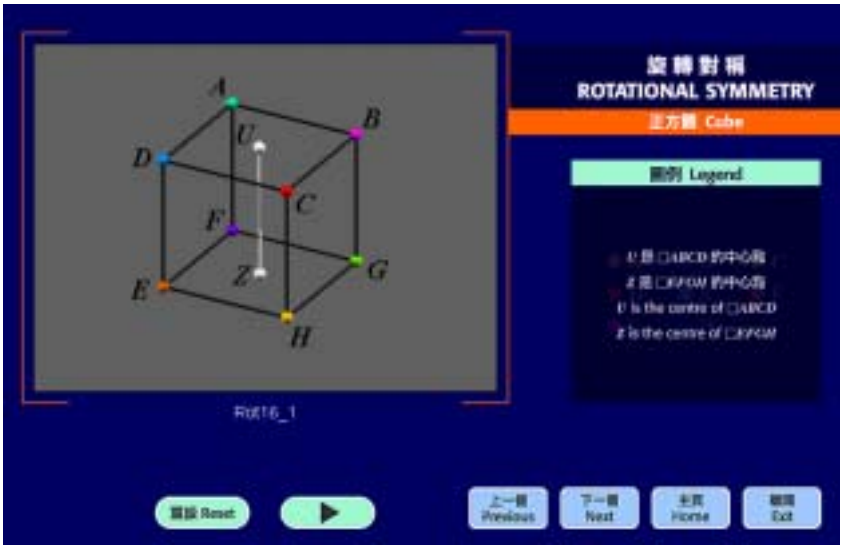

Fig. 2

3. The teacher further holds up a square and the cube. Students are guided to recognize the difference of symmetries of 2-D and 3-D figures:
   (a) In a 2-D figure which has reflectional symmetry, there is at least one line of reflection but a plane of reflection will be found in 3-D figures instead.
   (b) For the case of rotational symmetry, a 2-D figure will have a centre of rotation but a 3-D figure will have the axis of rotation.

4. Students are divided into groups and each group is provided with cubes and regular tetrahedra. The teacher then asks students to identify the symmetry elements found in the solids. Students are asked to locate:
   (a) All axes of rotation and count the number of them (for rotational symmetry).
   (b) All planes of reflection and count the number of them (for reflectional symmetry).

5. Students of different groups are invited to report to the whole class their findings of these symmetry elements. They are requested to demonstrate the axes of rotation and the planes of reflection to others. Students of other groups are invited to add more axes or planes if the numbers given are not correct. For some students who find it difficult to imagine the reflection or rotation, the teacher could use the provided computer files Ref_cube.exe, Ref_tetr.exe, Rot_cube.exe and Rot_tetr.exe to demonstrate the movement (See Fig. 3 to Fig. 6).

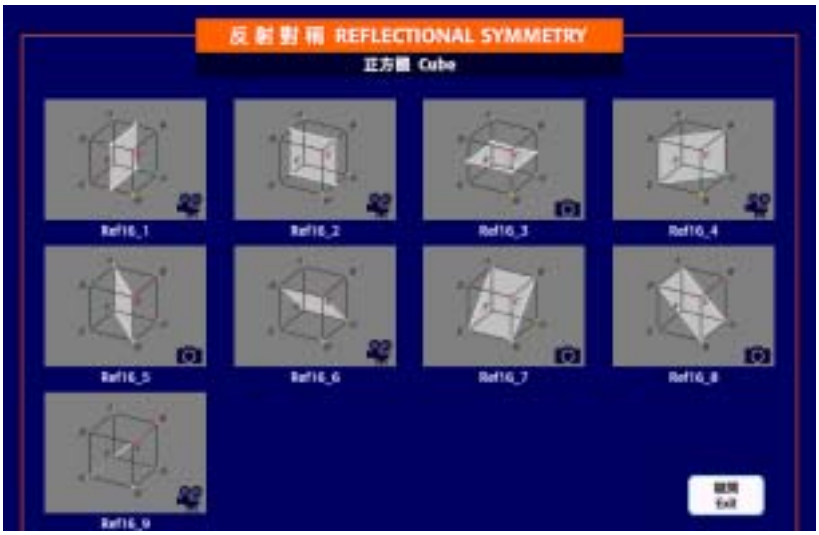

Fig. 3

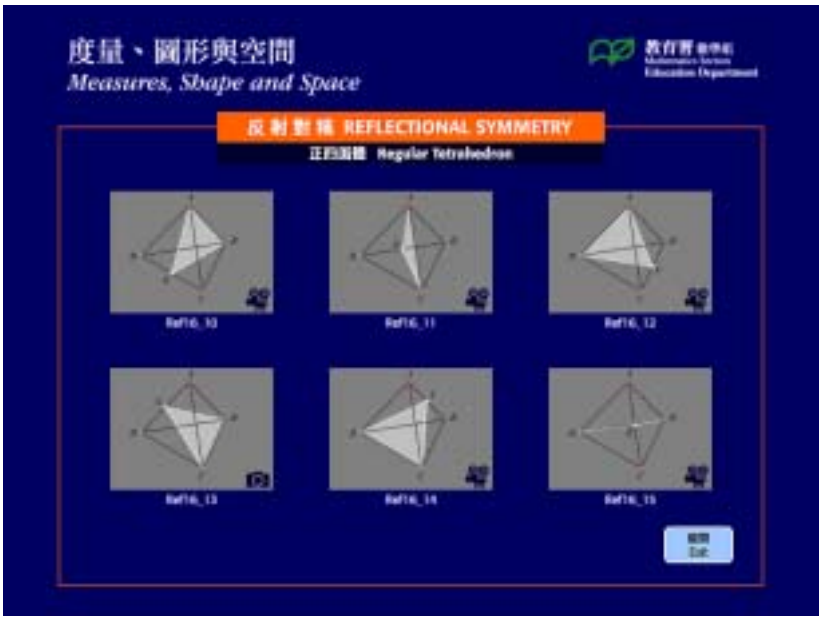

Fig. 4

Fig. 5

Fig. 6

6. The teacher then guides students to summarize the findings and conclude the idea of reflectional and rotational symmetries in 3-D figures.

Notes for Teachers:

1. The objective of the activity is to provide experiences for students to appreciate the symmetries in 3-D shapes. Cubes and regular tetrahedra are used as illustration because of their simplicity and availability. Through these, students could be able to identify the differences between symmetry in a polyhedron and that in a polygon. Teachers can get the pre-drawn polygons from Appendix B or the net of regular polyhedra from Appendix A.

2. The teachers can use rubber band to illustrate the **plane of reflection** for reflectional symmetry in a solid. For rotational symmetry in a solid, teacher can use straw in a cube to illustrate **the axis of rotation** (See Fig. 7).

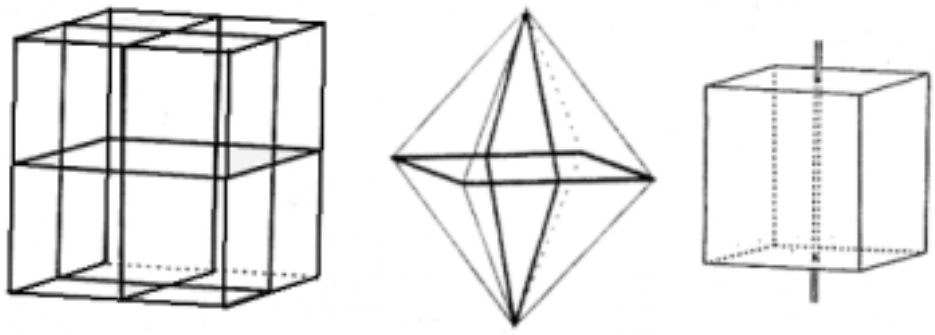

Fig. 7

3. The answers for the number of planes of reflection and the axes of rotation for cube and tetrahedron are as follows:

No. / Solid	Plane of reflection	Axis of rotation
Cube	9	13
Tetrahedron	6	7

The teacher should note that the number of folds for different axes of rotation is different. For cube, it has three 4-fold axes, four 3-fold axes, six 2-fold axes whereas tetrahedron has four 3-fold axes and only three 2-fold axes. However, students need not distinguish the numbers for different folds.

4. Teachers should note that some students may have difficulty to identify all the symmetry elements of the above two regular polyhedra. Therefore, the teacher may use the provided computer files to demonstrate the movement (See Fig. 8).

Fig. 8

Nevertheless, teachers can invite more able students to explore the symmetry elements for other regular polyhedra as project work. The numbers are as follows:

No. / Solids	Plane of reflection	Axis of rotation
Regular octahedron	9	13
Regular dodecahedron	15	31
Regular Icosahedron	15	31