



## EXEMPLAR 24 :

### Mid-point Theorem

**Objective:** To explore the mid-point theorem of triangles

**Key Stage:** 3

**Learning Unit:** Quadrilaterals

**Materials Required:** Dynamic geometry software such as *Geometer's Sketchpad* (later referred as *Sketchpad*)

**Prerequisite Knowledge:** Parallel lines, similar triangles

#### Description of the Activity:

1. The teacher briefly revises the properties of similar triangles to students and explains the objective of this activity.
2. The teacher distributes Worksheet to students. Students are asked to draw a triangle using computer and explore the mid-point theorem.
3. The teacher may ask students to present their findings to the whole class. The teacher gives the conclusion as the mid-point theorem.
4. The teacher asks students to form groups to suggest a formal proof to the theorem. The teacher may guide students to consider the simple proof by using similar triangles. Here are some suggested questions for discussion.
  - (a) What is the relation between  $\triangle ABC$  and  $\triangle ADE$ ? Can you use the given information to prove this relation?
  - (b) Can you find a relation between  $DE$  and  $BC$ ?
  - (c) Is  $DE$  parallel to  $BC$ ? Why?
  - (d) Can you find a relation between  $\angle ADE$  and  $\angle ABC$ ?
5. The teacher invites some students to present their formal proofs to the whole class and gives comments if necessary.

### Worksheet: Mid-point Theorem

1. Open a new *Sketchpad* file.
2. Draw a triangle and label it as  $\triangle ABC$ .
3. Construct the mid-points  $D$  and  $E$  of the sides  $AB$  and  $AC$  respectively.
4. Draw the line segment  $DE$  (see Fig.1).

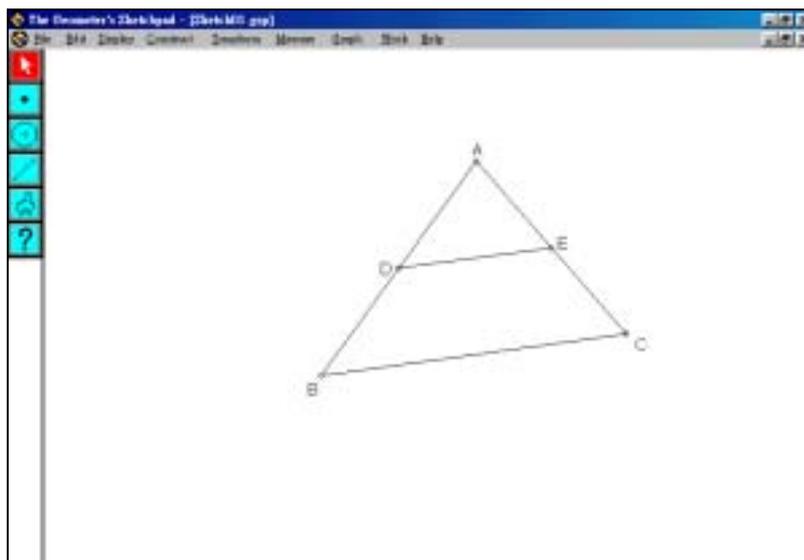


Fig.1

5. Drag the points  $A$ ,  $B$  and  $C$  respectively to observe the change. Record five sets of data in Table 1.

Set	$DE$	$BC$	$\angle ADB$	$\angle ABC$	$\angle AED$	$\angle ACB$
1						
2						
3						
4						
5						

Table 1

6. What is/are the relation(s) between  $DE$  and  $BC$ ? Write down your findings below.

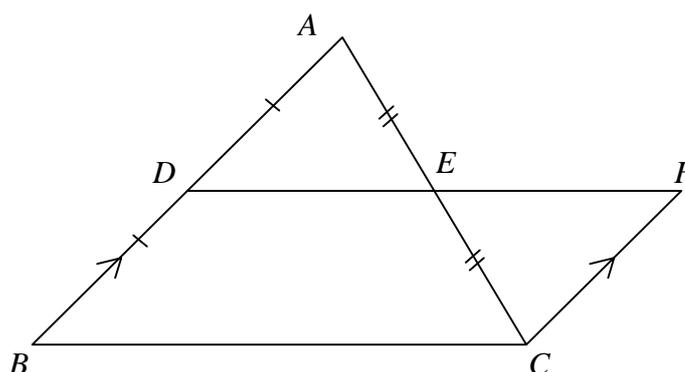
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**Notes for Teachers:**

1. The first part of this exemplar adopts an inquiring approach to build up the concept of mid-point theorem. Students are expected to explore the theorem by themselves.
2. The suggested proof of the theorem is using the theorem of "2 sides proportional, one included angle". The teacher may introduce another classic proof, which is outlined below. The teacher may further explain why this proof is different from that suggested in the activity.
  - (a) Draw a line through  $C$  parallel to  $BA$  and meet  $DE$  produced at  $F$ .
  - (b) Show that  $\triangle ADE \cong \triangle CFE$ .
  - (c) Deduce that  $BCFD$  is a parallelogram.
  - (d) Deduce that  $DE = \frac{1}{2}BC$  and  $DE \parallel BC$ .



**Operation Procedure:**

1. Select the **Segment** icon  to construct three sides of a triangle. Label the triangle as  $\triangle ABC$ .
2. Hold down the **Shift** key and click the segments  $AB$  and  $AC$ . Select **Construct | Point At Midpoint** in the pull-down menu to construct the two mid-points. Label it as point  $D$  and  $E$  respectively.
3. Select the **Segment** icon . Draw a line segment joining the points  $D$  and  $E$ .
4. To measure the lengths of  $DE$  and  $BC$ , hold down the **Shift** key. Click the segments  $DE$  and  $BC$ . Select **Measure | Length**. To measure  $\angle ADE$ , hold down the **Shift** key. Select points  $A$ ,  $D$  and  $E$  sequentially. Select **Measure | Angle**. Similar method to measure  $\angle ABC$ .
5. To find  $\frac{BC}{DE}$ , hold down the **Shift** key. Click the segments  $BC$  and  $DE$  sequentially. Select **Measure | Ratio**.