Exemplar 7: Using Different Properties to Construct a Square with Information Technology

Dimension : Measures, Shape and Space

Learning Unit : Quadrilaterals

Key Stage : 3

Materials Required : Dynamic Geometry Software such as *Geometer's Sketchpad* (later referred as *Sketchpad*)

Prerequisite Knowledge :
(1) Properties of special quadrilaterals such as parallelograms, rectangles, squares, rhombuses, kites and trapeziums.
(2) Basic concepts in transformation such as translation, reflection and rotation.
(3) Basic IT geometric construction skills.

Key Features :
This exemplar consists of two parts. Each part is in the Foundation Part of the *Syllabus* and illustrates the use of different built-in functions of the same software in constructing squares to cater for different learning abilities of students.

Part	Activities	Less able students	Average students	More able students
A	(I) Revision	✓	✓	
	(II) Demonstration	✓	✓	✓
B	(I) Constructing a square with guided instructions	✓	✓	
	(II) Exploring a new method		✓	
	(III) Constructing in their own methods.			✓

Table 1

Remark: ✓ represents the parts that can be participated by students when they start to learn the captioned topic.

Part A :

Part A(I) is a consolidation activity which allows students to review the properties of quadrilaterals previously learned. Part A(II) is a demonstration showing students the way to construct a square with the help of *Sketchpad*.

Part B :

This part is divided into three activities. With the help of software, students are asked to use other methods to construct a square and investigate the minimum number of steps required to draw a square.
In Activity I, students construct a square with guided instructions provided by the teacher.
In Activity II, students are required to further explore another method to draw a square. It is quite difficult for less able students and is not recommended to them.
In Activity III, students are expected to use what they know about the properties of squares to come up as many different ways as they can to construct a square without guiding instruction for the construction process. The level of difficulty is comparatively higher. A deductive proof for the findings is required.
A challenging problem is also included in Activity III for students' further investigation.

Description of the Activity:

Part A

(I) To review the properties of quadrilaterals

1. Distribute Worksheet 7.1A to the less able and average students or Worksheet 7.1B to average students and ask them to match the quadrilaterals with the properties listed.

2. Check answers with students and further discuss the following questions:
   (a) Which quadrilaterals have the properties of a parallelogram?
   (b) Which quadrilaterals have the properties of a rectangle?
   (c) Which quadrilaterals have the properties of a rhombus?

3. Ask students to classify the quadrilaterals into groups according to their own criteria.

4. Invite students to present their classification. Allow students to debate on their classification criteria. For example, some students may have different opinions on whether rhombuses are kites.

5. Use a Venn Diagram to illustrate and conclude the classification.

6. In drawing the conclusion, further discuss the minimum conditions to construct a square by applying the properties.

Worksheet 7.1A : Properties of Quadrilaterals

Put a tick in the appropriate boxes to match the following quadrilaterals with their corresponding properties.

Properties	Parallelogram	Rectangle	Square	Rhombus	Kite	Trapezium
1. Four right angles						
2. Opposite angles equal						
3. Consecutive angles supplementary						
4. Four sides equal						
5. Two distinct pairs of consecutive sides equal						
6. Opposite sides equal						
7. Opposite sides parallel						
8. One pair of sides parallel						
9. Diagonals equal						
10. Diagonals perpendicular						
11. Diagonals bisect each other						

Worksheet 7.1B : Properties of Quadrilaterals

Write down the relation between their sides, angles and diagonals (such as whether they are equal, bisect each other, etc) of the quadrilaterals:

1. Parallelogram

Sides	
Angles	
Diagonals	

2. Rectangle

Sides	
Angles	
Diagonals	

3. Square

Sides	
Angles	
Diagonals	

4. Rhombus

Sides	
Angles	
Diagonals	

5. Kite

Sides	
Angles	
Diagonals	

6. Trapezium

Sides	
Angles	
Diagonals	

(II) Construction of a Square (For teachers' demonstration)

1. Introduce the activity to students and revise some basic IT geometric construction skills such as constructing parallel lines, perpendicular lines, a circle etc.

2. Demonstrate two methods to construct a square.

<u>Method 1</u>:

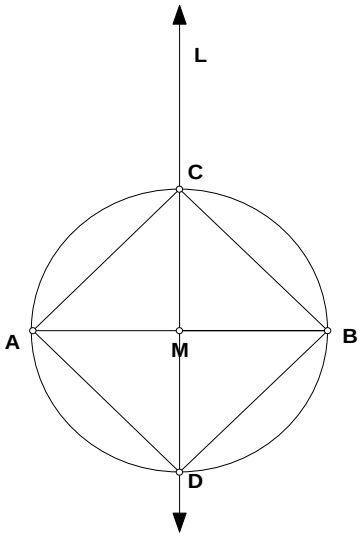
(i) Construct a segment AB and mark its mid-point M.

(ii) Construct a circle with the segment as its diameter. In this case the point M is the centre and AM is the radius.

(iii) Construct a line L through M and perpendicular to AB.

(iv) Mark the intersections of L and the circle as C and D.

(v) Join the four points A, D, B and C on the circle to get a quadrilateral.



<u>Method 2</u>:

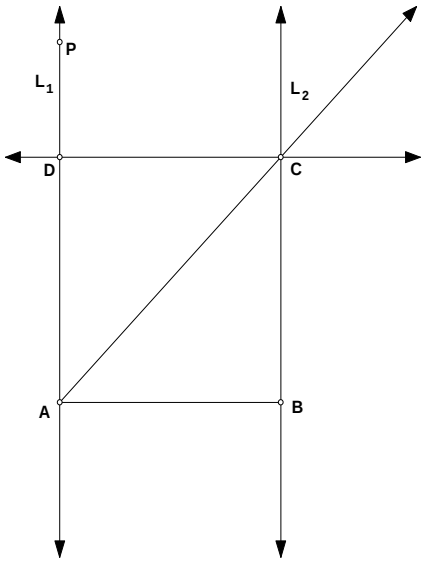
(i) Construct a line segment AB.

(ii) Construct two perpendicular lines L_1 and L_2 through its end points A and B.

(iii) Mark a point P on L_1.

(iv) Construct an angle bisector of angle PAB. This angle bisector will intersect L_2 at point C.

(v) Construct a line through C and perpendicular to L_2. This line intersects L_1 at point D. A square ABCD is then obtained.



3. Ask students whether the figure ADBC in Method 1(ABCD in Method 2) is a square and justify their assertion with reasons.

4. Guide students to observe that not all the properties of a square are needed in the construction and recall the idea of minimum conditions.

Part B

(I) For all students,

1. Select two methods below and ask students follow the procedures to construct a square:

Method 3:

(i) Construct a line segment AB.

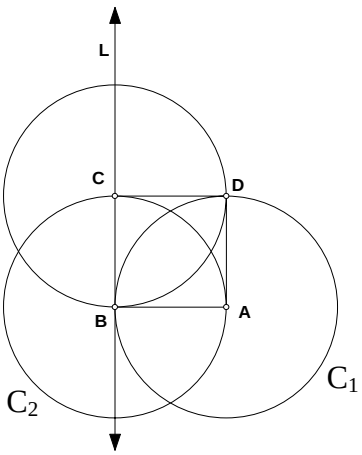
(ii) Construct two circles C_1 and C_2 centred at A and B respectively with radius BA.

(iii) Construct a line L at B and perpendicular to BA.

(iv) Mark the intersection of the circle C_2 and Line L as point C.

(v) Construct a circle centred at C with radius BA.

(vi) This circle will intersect C_1 at point D. Then ABCD is a square.



Method 4:

(i) Construct a line segment BA.

(ii) Construct a circle centred at A with radius AB.

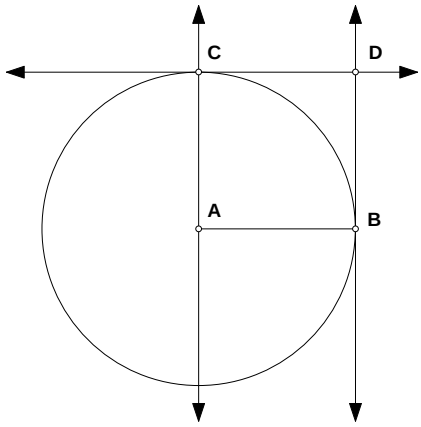
(iii) Construct a line through A and perpendicular to AB.

(iv) Mark the intersection of the circle and this line as point C.

(v) Construct a line through C and parallel to AB.

(vi) Construct a line through B and parallel to AC.

(vii) The point of intersection of the lines drawn in step (v) and (vi) is labeled as D. Then ABDC is a square.



Method 5:

(i) Construct a line segment AB.

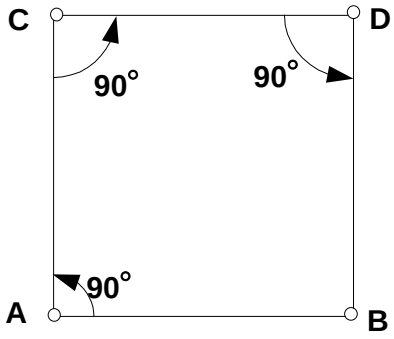
(ii) Rotate the segment 90º in the anti-clockwise direction about point A.

(iii) Mark the end point of the resulting segment as C.

(iv) Rotate AC 90º in the anti-clockwise direction about point C.

(v) Mark the end point of the resulting segment as D.

(vi) Rotate CD 90º in the anti-clockwise direction about point D. Then ABDC is a square.



Method 6:

(i) Construct a line segment AB.

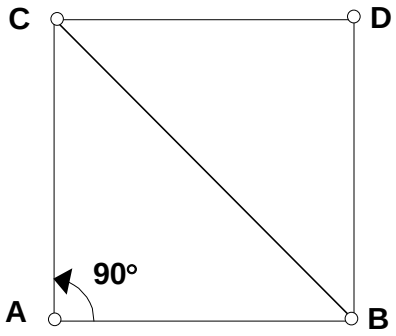
(ii) Rotate the segment 90º in anti-clockwise direction about point A.

(iii) Mark the end point of the resulting segment as C.

(iv) Construct segment BC.

(v) Reflect point A about BC.

(vi) Label the image as D. Then ABDC is a square.



2. Discuss with students which method is most efficient (i.e. the one with the fewest step).

3. Discuss with students the properties used in each construction of the square. Complete Worksheet 7.2A.

(II) For average students

1. Ask students to construct a square using their own methods.
2. Invite students to present their findings.
3. Discuss with students whether the quadrilaterals constructed by their methods are square and the reason.

(III) For able students,

1. Ask students to explore different methods of constructing a square.
2. Distribute Worksheet 7.2B to students to record the description of each method and the properties of the square applied in the construction.
3. Invite students to present their findings.
4. Discuss with students which method is most efficient and whether the figure is a square.
5. Pose the following challenging problem to students:
   As squares are rectangles, can you construct a square starting from any given rectangle? How?
   Ask students to give reasons in written form to support their constructions.

Worksheet 7.2A : Identify the properties used in the constructions of squares.

Complete the table.

Method	Properties of squares	Remarks
1		Demonstrated by the teacher in Part A(II).
2		
3		Guided construction
4		
5		
6		

Worksheet 7.2B : Construction of a Square

Record the construction procedures of each method to construct a square and the properties used in the constructions.

Method 1 :

Properties:

Method 2 :

Properties:

Method 3 :

Properties:

Method 4 :

Properties:

Notes for Teachers:

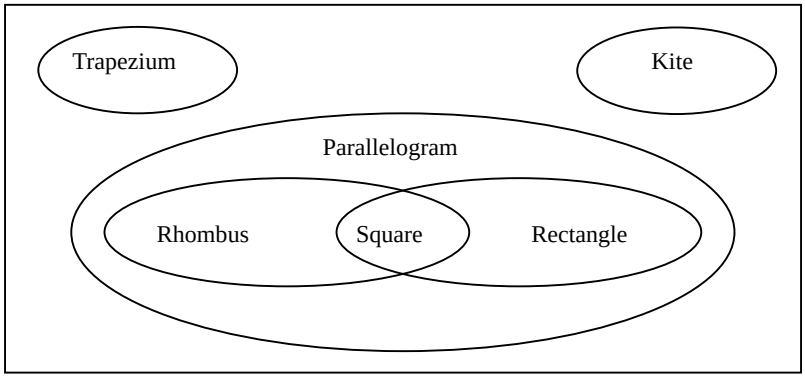
Part A :

1. Properties of the six special quadrilaterals:

Properties	Parallelogram	Rectangle	Square	Rhombus	Kite	Trapezium
1. Four right angles		✓	✓			
2. Opposite angles equal	✓	✓	✓	✓		
3. Consecutive angles supplementary	✓	✓	✓	✓		
4. Four sides equal			✓	✓		
5. Two distinct pairs of consecutive sides equal					✓	
6. Opposite sides equal	✓	✓	✓	✓		
7. Opposite sides parallel	✓	✓	✓	✓		
8. One pair of sides parallel						✓
9. Diagonals equal.		✓	✓			
10. Diagonals perpendicular			✓	✓	✓	
11. Diagonals bisect each other	✓	✓	✓	✓		

2. (a) Squares, rhombuses and rectangles are all parallelograms.
   (b) Squares have the properties of a rectangle.
   (c) Squares have the properties of a rhombus.

3. The following Venn Diagram can be used to illustrate the classification.



4. Teachers can use the enclosed *Sketchpad* Scripts Method1.gss and Method2.gss to demonstrate methods 1 and 2 respectively.

Part B:

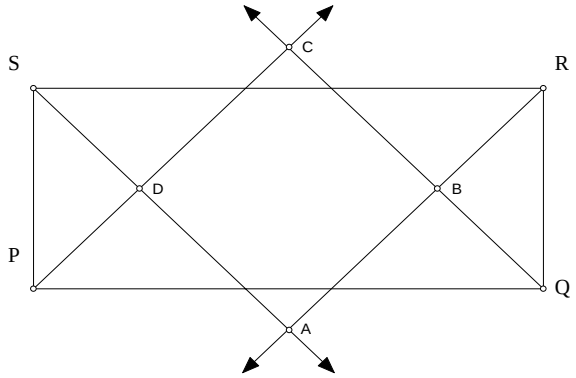
1. Properties of squares used in each suggested construction procedure are listed in the table.

Method	Properties of square	Remarks
1	Two diagonals are equal, perpendicular and bisect each other.	The first four methods illustrate the use of straight edges and compasses in the construction of a square.
2	Two right angles and the diagonals bisect opposite angles.	
3	Four equal sides with one right angle.	
4	Opposite sides are parallel and adjacent sides are equal and perpendicular to each other.	
5	Four right angles and four equal sides	The last two methods illustrate the use of transformation in the construction since a square has both reflectional and rotational symmetry.
6	Two equal sides, one included right angle and reflectional symmetry about the diagonal.	

2. A square may be obtained from the intersection of four angle bisectors of a rectangle. In the figure below, ABCD is a square. Students may not consider this approach to obtain a square from a rectangle. The teacher may need to give some hints to students such as constructing the angle bisectors of the rectangle.

3. Teachers can use the enclosed *Sketchpad* scripts Method3.gss, Method4.gss, Method5.gss and Method6.gss respectively for demonstrating methods 3 to 6. Or teachers can use the attached construction procedures to construct the diagrams.

4. A deductive proof can be requested for more able students.



Proof: Since $\angle PSD = \angle DPS = 45°$,

$\angle PDS = 90°$ ($\angle$s sum of a Δ)

$\therefore \angle ADC = 90°$ (vert. opp. $\angle$s)

Similarly $\angle DAB = \angle ABC = \angle BCD = 90°$

Also it can be shown that $\Delta SPD \cong \Delta QRB$. (A.S.A.)

$\therefore PD = DS = BR = BQ$ .....(*) (corr. sides of congruent Δs)

As $\angle CPQ = \angle CQP$, $PC = CQ$ .....(**) (base $\angle$s equal)

Thus $CD = BC$ (from * and **)

Hence ABCD is a square.

Suggested Operation Procedure for the demonstration (Based on *Sketchpad* 3.0) :

Method 1

1. Click the **Segment tool**  and drag to draw a horizontal segment.

2. Click the **Label** tool  and label the endpoints of the segment as A and B.

3. Select the segment AB. Go to the **Construct** menu and choose **Point At Midpoint**.

4. Click the Label tool and give a label to the midpoint. Double click the label and rename it as M.

5. Click on the **Selection Arrow** tool  Hold down the **Shift** key.
   Select points A and M. Go to the **Construct** menu and choose **Circle By Centre And Point**.

6. Hold down the **Shift** key. Select point M and segment AB. Go to the **Construct** menu and choose **Perpendicular Line** to construct a line through M and perpendicular to AB.

7. Click the **Label** tool and label the line as L.

8. Click on the **Selection Arrow** tool. Hold down the **Shift** key. Select the circle and line L. Go to the Construct menu and choose **Point At Intersection**.

9. Click the **Label** tool and label the points of intersection as C and D.

10. Click on the **Selection Arrow** tool. Hold down the **Shift** key. Select the points A, B, C and D. Go to the **Construct** menu and choose **Segment**. Then a square ADBC is obtained.

Method 2

1. Repeat step 1 and 2 in Method 1.

2. Click on the **Selection Arrow** tool. Hold down the **Shift** key. Select point A and segment AB. Go to the **Construct** menu and choose **Perpendicular Line** to construct a line through A and perpendicular to AB.

3. Hold down the **Shift** key. Select point B and segment AB. Go to the **Construct** menu and choose **Perpendicular Line** to construct a line through B and perpendicular to AB.

4. Click the **Label** tool and label the perpendicular lines at A and B as L_1 and L_2 respectively.

5. Click on the **Point** tool and mark a point P on L_1.

6. Click on the **Selection Arrow** tool. Hold down the **Shift** key. Select points P, A and B. Go to the **Construct** menu and choose **Angle Bisector**.

7. Hold down the **Shift** key. Select the angle bisector and L_2. Go to the **Construct** menu and choose **Point At Intersection**.

8. Label this point as C.

9. Click on the **Selection Arrow** tool. Hold down the **Shift** key. Select point C and L_2. Go to the **Construct** menu and choose **Perpendicular Line**.

10. Mark the point of intersection of the perpendicular line drawn in step 9 and L_1.

11. Label this point as D. Then a square ABCD is obtained.

Other operation procedure involving in methods 3 to 6:

(I) *Construct a circle centre at point A and with radius AB*

Click on the **Selection Arrow** tool. Hold down the **Shift** key. Select points A and B. Go to the **Construct** menu and choose **Circle By Centre And Point**.

(II) Rotate a line segment AB about the point A by 90º

1. Click on the **Selection Arrow** tool. Select point A. Go to the **Transform** menu and choose **Mark Centre**. Then point A is the centre of rotation.
2. Hold down the **Shift** key. Select segment AB and point B. Go to the **Transform** menu and choose **Rotate**. Enter **90º** in the box and click **OK**.

(III) Reflect a point A about a line segment BC

1. Click on the **Selection Arrow** tool. Select segment BC. Go to the **Transform** menu and choose **Mark Mirror**. Then BC is the line of reflection.
2. Select point A. Go to the **Transform** menu and choose **Reflect**. Then an image of A is obtained.