

2.2: The Lorenz Curve

Objectives:

After the lessons, students will be able to

1. identify the boundaries of a Lorenz Curve;
2. understand the presentation of income inequality in a Lorenz Curve;
3. understand the relationship between the change in income inequality and the shift of the Lorenz Curve;
4. point out the major disadvantage of using a Lorenz Curve to compare income inequality between countries. (for more able students)

Prerequisite knowledge:

1. An understanding of mapping information from a graph.
2. An understanding of the limitation of measuring income inequality with the percentages of total income earned by the lowest and highest-income groups.

Time allocation:

40-50 minutes

Learning Activities:

Three learning activities

2.2 The Lorenz Curve

Activity 1: Boundaries of a Lorenz Curve

A. Objective: To understand the boundaries of a Lorenz Curve in general.

B. Time required: 12 minutes

C. Materials needed: two tables of income distribution (TM1-2)

D. Teaching plan and instructional procedures:

Teacher Activities	Student Activities	Time (mins)	Target Content/ Skills	Remarks
1. Explain the meaning of cumulative percentage of total income and households.(TM1)	Listen and jot down notes.	1	Review and apply the learning of last section.	
2. Ask students to consider the relationship between two variables when all households have equal income.	Think and discuss in pairs.	2	Making prediction with reasons.	<ul style="list-style-type: none"> • An excel programme was uploaded to the website of EDB so that teacher can generate Lorenz Curves in class. • Teacher may generate a Line of Equality with the information from TM1, and a special Lorenz Curve (all income earned by the
3. Show a Lorenz Curve (Line of Equality) which indicates equal distribution of income. Ask students to find the missing value in Table 1.	Determine the missing values in Table 1 either individually or in pairs.	2	Understand the upper boundary of the Lorenz Curve and map data from graph.	
4. Define Line of Equality.	Listen and jot down notes.	1	Understand the meaning of perfect equality and the corresponding shape of the Lorenz Curve.	
5. Show a Lorenz Curve indicating extreme inequality (TM2) and ask students to	Determine the missing values in Table 2 either individually or in pairs.	2	Understand the lower boundary of the Lorenz Curve	

determine the income distribution and the missing value in Table 2.			and map data from graph.	richest 20%) with the information from TM2.
6. Show a Lorenz Curve showing extreme inequality.	Listen and jot down notes.	1	Learn the shape of the Lorenz Curve showing extreme inequality.	
7. Ask students where a Lorenz Curve would lie in general and why.	Discuss in pairs.	2	Speculate and predict the position of a Lorenz Curve based on its boundaries.	
8. Conclude the position and shape of a Lorenz Curve in general and give assignment.	Listen and try to sketch some Lorenz Curves.	1	Confirm speculation and reflect on the process of prediction.	Assignment 1.

E. Teaching Materials

Teaching Material 1 (TM1)

Quintile	Households		Income	
	Percentage (%)	Cumulative Percentage (%)	Percentage (%)	Cumulative Percentage (%)
Lowest 20%	20	20	20	20
Second 20%	20	40	20	40
Third 20%	20	60	20	60
Fourth 20%	20	80	20	80
Highest 20%	20	100	20	100

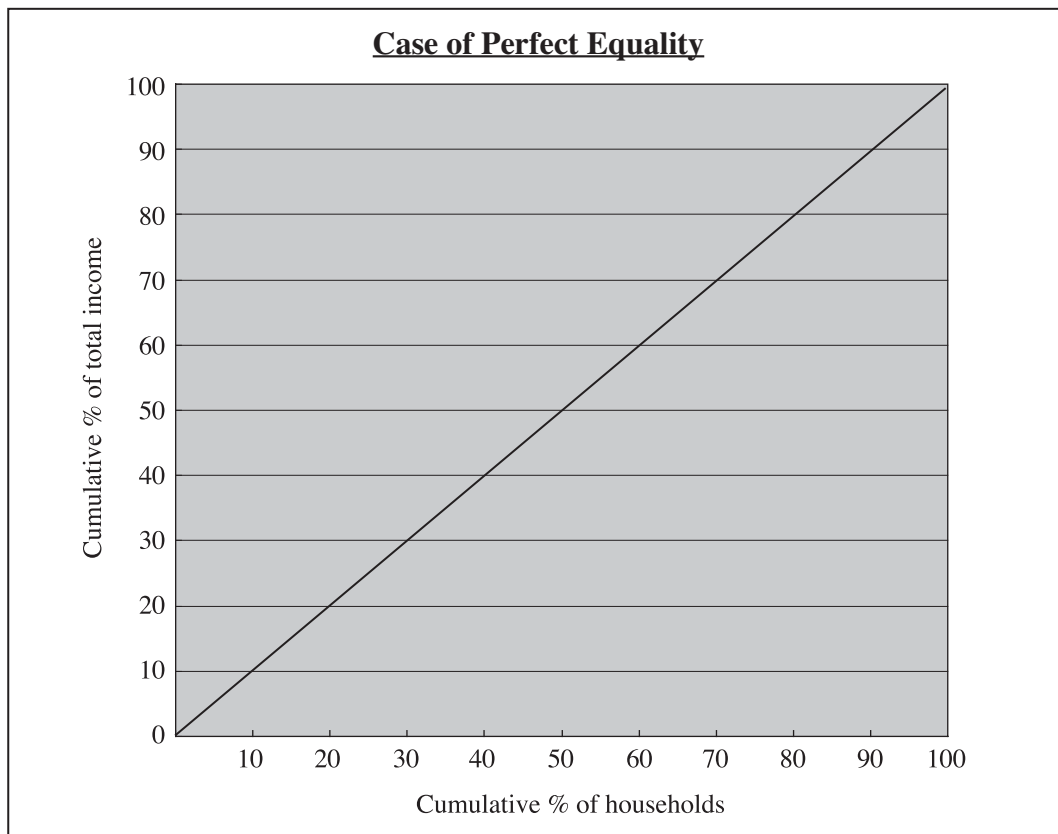


Table 1: The cumulative percentages of households and total income of 3 points on Line of Equality (to be done by students)

Point on Line of Equality	Cumulative percentage of households	Cumulative percentage of total income
Point 1	15%	15%
Point 2	48%	48%
Point 3	90%	90%

Teaching Material 2 (TM2)

(Note: this Lorenz Curve shows all incomes earned by the richest 20%)

Quintile	Households		Income	
	Percentage (%)	Cumulative Percentage (%)	Percentage (%)	Cumulative Percentage (%)
Lowest 20%	20	20	0	0
Second 20%	20	40	0	0
Third 20%	20	60	0	0
Fourth 20%	20	80	0	0
Highest 20%	20	100	100	100

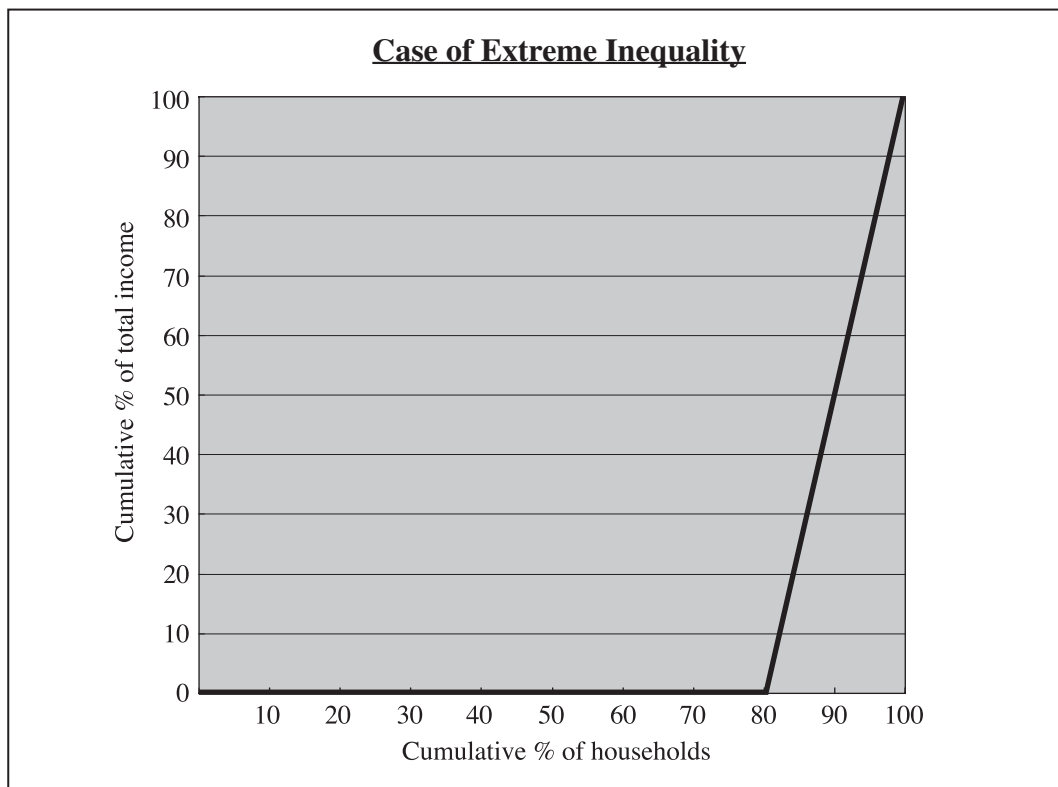
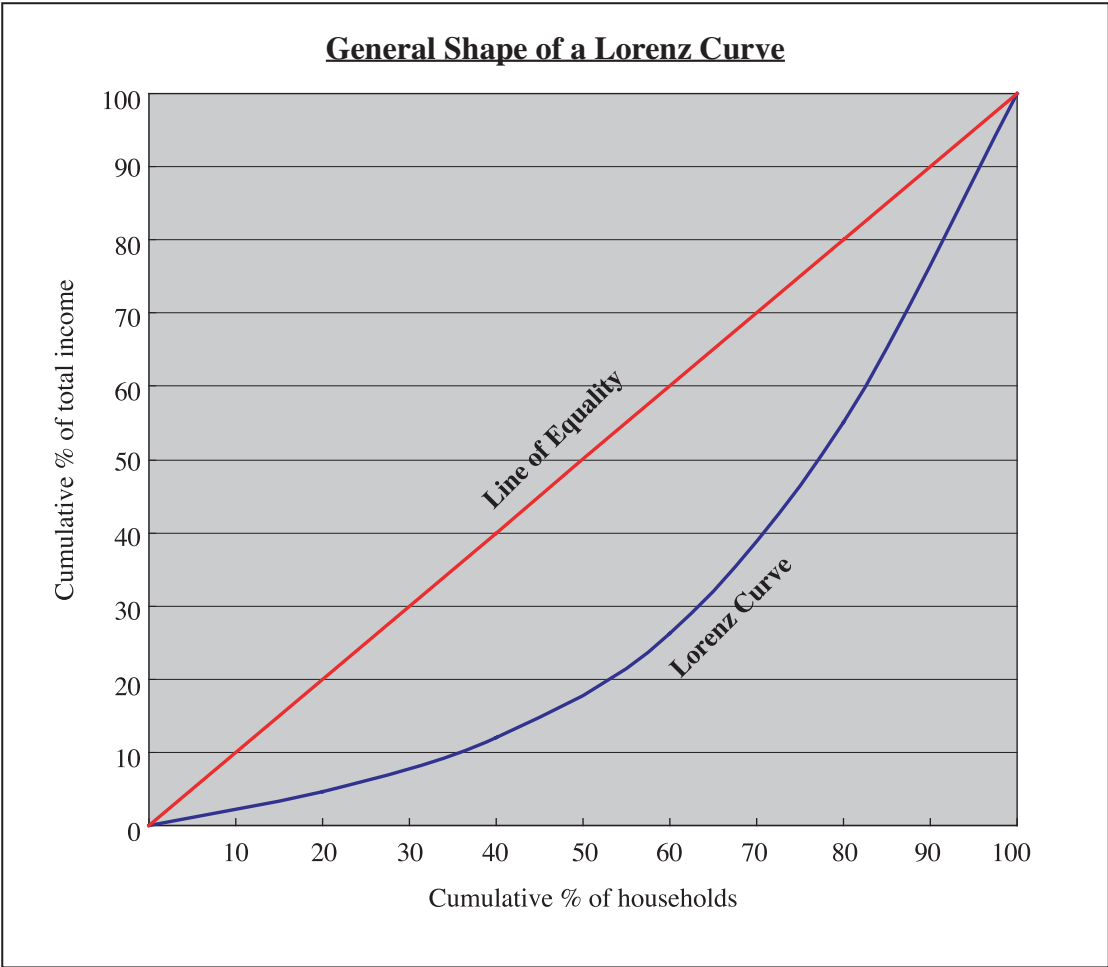


Table 2: The cumulative percentages of households and total income of 3 points on this particular Lorenz Curve (to be done by students)

Point on Lorenz Curve	Cumulative percentage of households	Cumulative percentage of total income
Point 1	15	0
Point 2	48	0
Point 3	90**	50

**** Assuming all incomes are evenly distributed among households of the highest 20%**



F. Assignment

Assignment 1

Suppose the total incomes of both countries A and B are \$5000.
 In Country A, all households earn equal income.
 In Country B, the richest households (20%) earn all the income.

- a. Fill in the following table to show
- I. the percentages of total income earned by different income groups, and
 - II. the cumulative percentages of total income earned by different income groups.
- b. Sketch the Lorenz Curves of these countries and label the axes clearly.

Income Group	Country A		Country B	
	Percentage of total income earned (%)	Cumulative percentage of total income earned (%)	Percentage of total income earned (%)	Cumulative percentage of total income earned (%)
Lowest 20%	20	20	0	0
Second 20%	20	40	0	0
Third 20%	20	60	0	0
Fourth 20%	20	80	0	0
Highest 20%	20	100	100	100

Conclusion:

In general, the Lorenz Curve of a country lies under the *line of equality*. When the income distribution is more even, the Lorenz Curve will be *closer* to the line of *equality*. When the income distribution is more unequal, the Lorenz Curve will be *farther away from* the line of *equality*.

2.2 The Lorenz Curve

Activity 2 : The relationship between Income Distribution and Lorenz Curve

A. Objectives:

- To relate the position of a Lorenz Curve to income inequality.
- To relate the change of income distribution to the change of the position of a Lorenz Curve.

B. Time required: 25 minutes

C. Materials needed: One page of data for generating a Lorenz Curve, and TM1-2

D. Teaching plan and instructional procedures:

Teacher Activities	Student Activities	Time (mins)	Target Content/ Skills	Remarks
1. Show three Lorenz Curves and ask “What do you think about the income inequality among these three countries?”	Work in pairs to discuss and propose a prediction.	3	Figure out the income inequality according to the positions of the Lorenz Curves.	<ul style="list-style-type: none"> • Generate three Lorenz Curves using data from TM1 and the excel programme. • Teacher may revise Table 1 according to the needs of students.
2. Ask students to find the missing values in Table 1.	Work in pairs find the missing values and the cumulative percentage of total income.	3	Map data from graphs.	
3. Ask students to explain the income inequalities and positions of Lorenz Curves according to the information of Table 1.	Discuss and explain the income inequalities among three countries in groups of four.	4	Confirm the income inequalities with information of Table 1, and the positions of Lorenz Curves.	
4. Feedback and comment on students’ answers and give assignment.	Listen and learn. Finish Assignment 1 at home or in class.	3	Understand the relationship between the position of a Lorenz Curve and income inequality.	

5. Go over the events in TM2 briefly and ask students to discuss the change of income inequality, and hence the shift of the Lorenz Curve.	Form groups of four, and each group discusses one event for 4 mins.	5	Identify the change in income inequality and relate such change to the shift of the Lorenz Curve.	
6. Invite one representative from each of the four groups to report.	Listen and think.	5	Conclude the relationship between the change in income inequality and the shift of the Lorenz Curve.	
7. Feedback, comment, and give assignment.	Listen and reflect on determining the relationship. Finish Assignment 2.	2		

E. Teaching Materials

Teaching Material 1 (TM1)

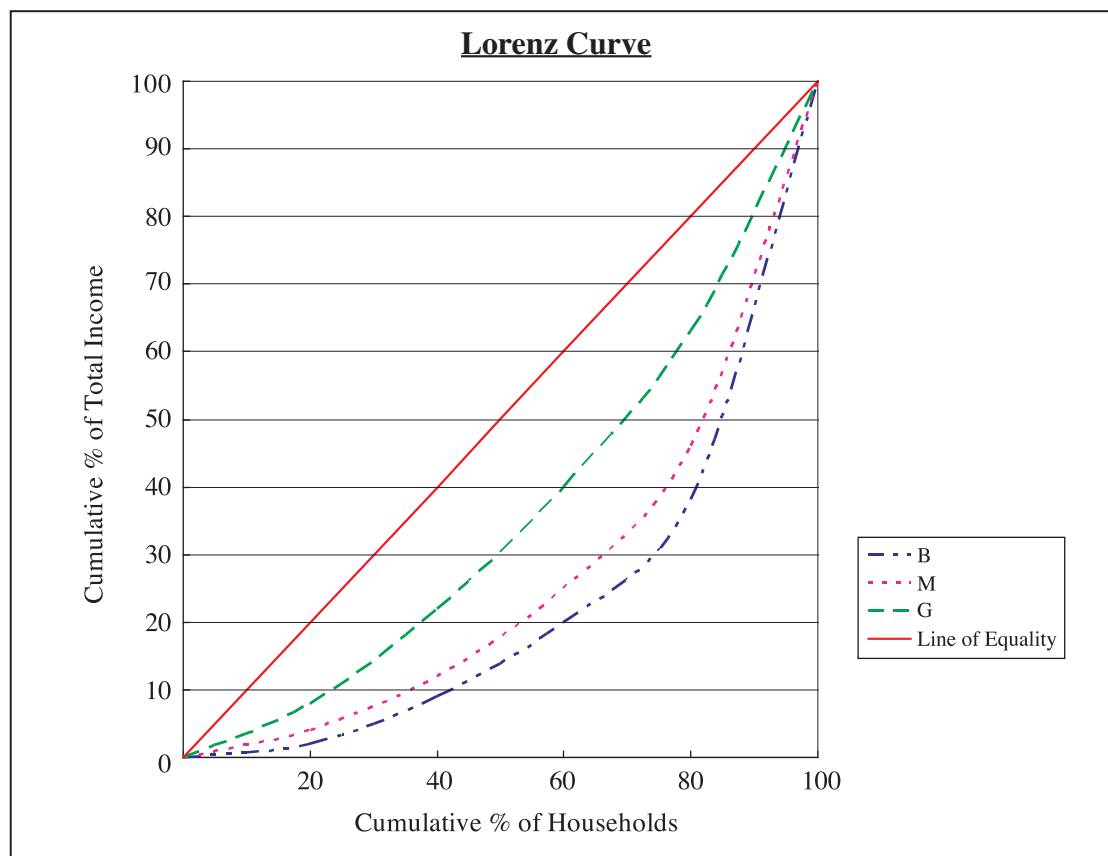
The following table shows the income distribution of countries B, M and G. (B, M and G are Brazil, Malaysia and Germany)

Fill in the missing values in the following table.

Income group	B (2003)		M (1997)		G (2000)	
	Percentage of income (%)	Cumulative percentage of income (%)	Percentage of income (%)	Cumulative Percentage of income (%)	Percentage of income (%)	Cumulative Percentage of income (%)
Lowest 20%	3	3	4	4	8	8
Second 20%	6	9	8	12	14	22
Third 20%	11	20	13	25	18	40
Fourth 20%	18	38	21	46	23	63
Highest 20%	62	100	54	100	37	100

For convenience, all values are in whole numbers.

Source: World Development Indicator 2006, World Bank



Teaching Material 2 (TM2)

Table 1 shows the income distribution of a country under the following conditions:

- A. \$2,000 of income of the highest-income group is transferred to the lowest-income group.
- B. Every household earns 10% more.
- C. All households have an additional annual income of \$2000.
- D. All households have to pay a lump sum annual tax of \$1000.

Income group	Total annual income		A Income transfer from the highest-income groups		B 10% additional income		C \$2000 additional income		D Lump sum tax of \$1000	
	Income earned (\$)	* (%)	Income earned (\$)	* (%)	Income earned (\$)	* (%)	Income earned (\$)	* (%)	Income earned (\$)	* (%)
Lowest 20%	2 000	3	4 000	7	2 200	3	4 000	6	1 000	2
Second 20%	7 000	15	7 000	20	7 700	15	9 000	19	6 000	13
Third 20%	10 000	32	10 000	37	11 000	32	12 000	36	9 000	29
Fourth 20%	16 000	58	16 000	62	17 600	58	18 000	61	15 000	56
Highest 20%	25 000	100	23 000	100	27 500	100	27 000	100	24 000	100
Total	60 000		60 000		66 000		70 000		55 000	
<i>Change in distribution</i>			More even		No change		More even		Less even	
<i>Shift of Lorenz Curve</i>			To the left		No change		To the left		To the right	

Table 1

* Cumulative percentages of income earned (%) are given to show the change of income distribution more clearly. The percentages are rounded up to whole numbers and the teacher may omit this column for more able students.

Table 2: The relationship between the change in income inequality and the shift of the Lorenz Curve.

Event	Change in income inequality	Explanation for the change	Shift of the Lorenz Curve
A. Income of the highest-income groups is transferred to the lowest-income groups.	More equal, or less unequal	The percentage of total income of the highest-income group decreases and that of the total income of the lowest-income group increases.	Shift to the left, closer to the Line of Equality.
B. Every household earns 10% more.	No change	The percentages of total income of different income groups do not change.	Lorenz curve does not shift.
C. All households have an additional annual income of \$2000.	More equal	The percentage of total income of high-income groups drops and the percentage of total income of low-income groups grows.	Shift to the left, closer to the Line of Equality.
D. All households have to pay a lump sum annual tax of \$1000.	Less equal	The percentage of total income of high-income groups increases and the percentage of total income of low-income groups decreases.	Shift to the right, farther away from the Line of Equality.

F. Assignments

Assignment 1

The following table shows the income distribution of Hong Kong in 1971 and 2001.

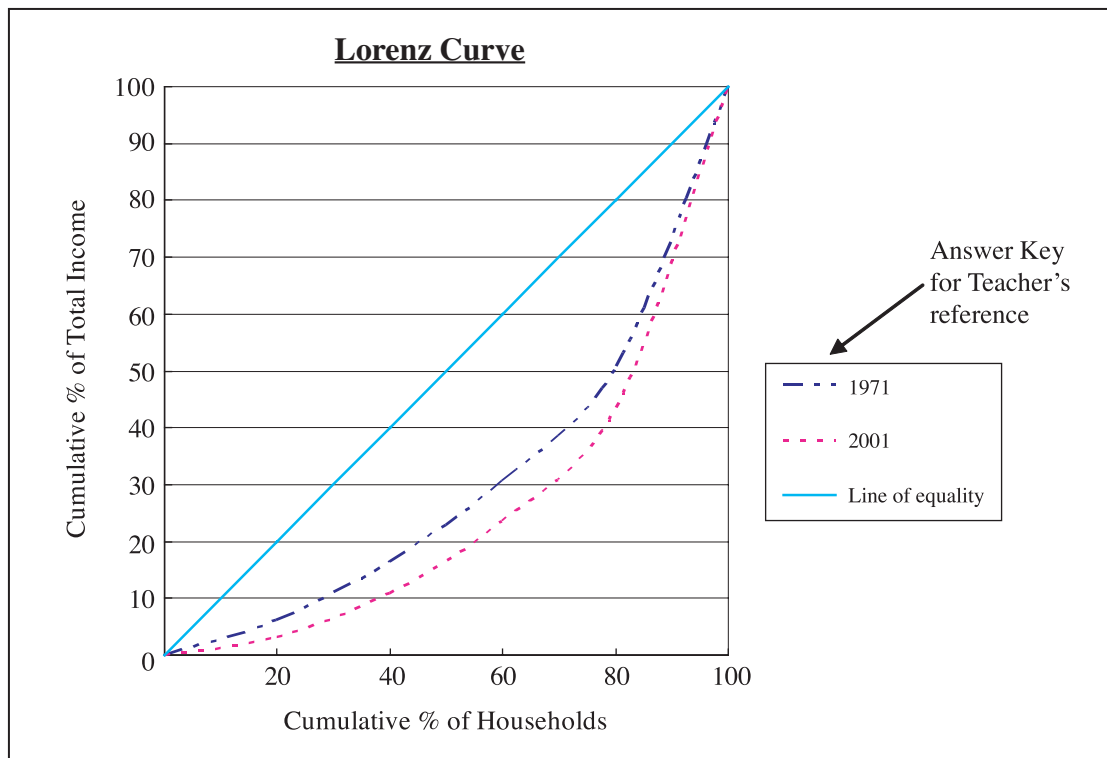
1. Explain how the income distribution changed.
2. In the following diagram, the two Lorenz Curves show the income distribution of Hong Kong in 1971 and 2001. Identify with reasons which curve shows the income distribution of 2001.

Distribution of Household Incomes in Hong Kong (1971-2001)

Unit: % of total income

Income group	Cumulative percentage of total income 1971	Cumulative percentage of total income 2001
Lowest 20%	6.2	3.2
Second 20%	16.4	11
Third 20%	30.7	23.6
Fourth 20%	50.7	43.5
Highest 20%	100	100

Source: Hong Kong Census and Statistics Department

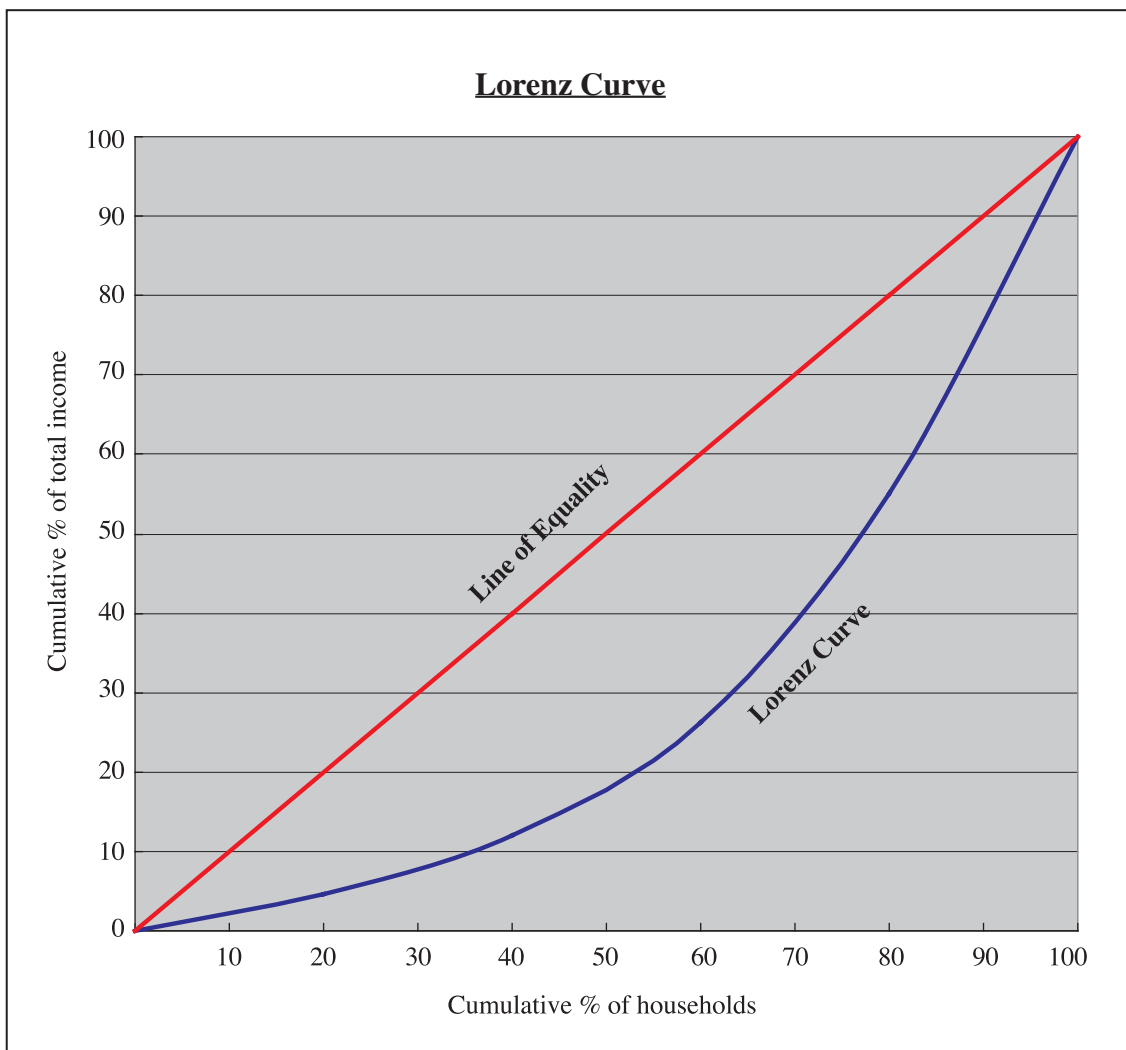


Teachers can also generate the curves using the excel programme provided.

Assignment 2

How would the following events/cases change the income distribution of a country?
Illustrate your answers with suitable Lorenz Curves.

The following Lorenz Curve can serve as an original situation. Students can analyze the following cases based on this graph.



- Some income of the low-income groups is transferred to the high-income groups.
- Government gives a fixed lump sum subsidy of \$500 to the low-income groups.
- Every household earns 5% less.
- All households have an additional annual income of \$1000.
- All households have to pay a lump sum annual tax of \$500.

Remarks:

Cases c, d and e may be suitable for more able students.

2.2 The Lorenz Curve

Activity 3 : Disadvantage of using the Lorenz Curve

(for more able students)

- A. Objective:** To learn the major disadvantage of using the Lorenz Curve.
- B. Time required:** 10 minutes
- C. Materials needed:** One page of income distribution data of countries X and Y (TM1)
- D. Teaching plan and instructional procedures:**

Teacher Activities	Student Activities	Time (mins)	Target Content/ Skills	Remarks
1. Show the income distribution of two countries (TM1). Ask students to find the percentages of income earned by the highest-income groups and compare the income distribution of these two countries.	Work in pairs to find the percentages of income earned by the highest-income groups and compare the income distribution of the two countries.	4	<ul style="list-style-type: none"> Calculate the percentages of income earned by the highest-income groups. Compare the income distribution by inspecting the cumulative percentages of total income earned by different income groups. 	<ul style="list-style-type: none"> Generate the Lorenz Curves with the information from TM1 and the excel programme provided.
2. Show the Lorenz Curves of countries X and Y.	Relate Lorenz Curves to the income distribution of the two countries.	2	Relate the income distribution to the Lorenz Curve.	
3. Ask students to explain the relationship of income distribution and Lorenz Curves based on the data of countries X and Y.	Discuss in pairs.	2	Explore the major disadvantage of using the Lorenz Curve.	
4. Explain the major disadvantage of using the Lorenz Curve and give assignment.	Listen and jot down notes. Finish Assignment 1.	2	Confirm the major disadvantage of using the Lorenz Curve.	

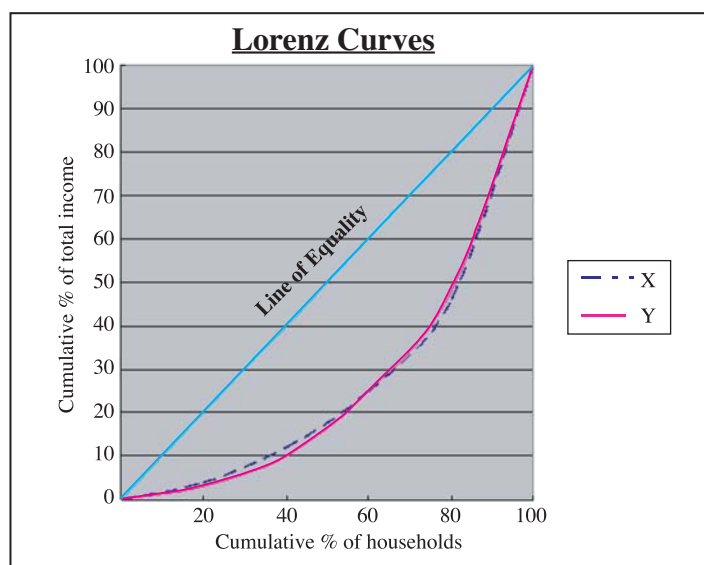
E. Teaching Materials

Teaching Material 1 (TM1)

The Lorenz Curves of Countries X and Y (2000) intersect.

Quintile	Country X (2000)		Country Y (2000)	
	Percentage of income (%)	Cumulative % of income (%)	Percentage of income (%)	Cumulative % of income (%)
Lowest 20%	4	4	3	3
Second 20%	8	12	7	10
Third 20%	13	25	15	25
Fourth 20%	21	46	23	48
Highest 20%	54	100	52	100

***The teacher may use the excel programme provided to generate two Lorenz Curves.**



The difference in income distribution between countries X and Y is not clear. Though the low-income groups of country X have a higher percentage of total income, representing a more even income distribution, the highest-income group of country X has higher percentage of total income, representing a less even income distribution. Furthermore, as the Lorenz Curves of these countries intersect, it is hard to tell which country has greater income inequality.

F. Assignment (For more able students)

What are the pros and cons of using Lorenz Curve to show income inequality?

Possible Answers:

One advantage of using Lorenz Curve to show income inequality is that the ***shape and the position of the curve*** can indicate the income inequality well. A disadvantage is that when the Lorenz Curves of two countries intersect, it would be difficult to judge the ***the difference of income distribution in that case.***